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PATTERNS OF PARADOX

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§1. A language of paradox. We begin with a propositional language L_P containing conjunction ($\wedge$), a class[1] of sentence names $\{S\alpha\}_{\alpha\in A}$, and a falsity predicate F. We (only) allow unrestricted infinite conjunctions, i.e., given any non-empty class of sentence names $\{S_\beta\}_{\beta\in B}$,

$$\wedge\{F(S_\beta) : \beta \in B\}$$

is a well-formed formula (we will use *WFF* to denote the set of well-formed formulae).[2]

The language, as it stands, is unproblematic. Whether various paradoxes are produced depends on which names are assigned to which sentences. What is needed is a denotation function:

$$\delta : \{S_\alpha\}_{\alpha\in A} \rightarrow WFF.$$

For example, the L_P sentence "$F(S_1)$" (i.e., $\wedge\{F(S_1)\}$), combined with a denotation function δ such that$\delta(S_1)$ = "$F(S_1)$", provides the (or, in this context, a) *Liar Paradox*.

To give a more interesting example, *Yablo's Paradox* [4] can be reconstructed within this framework. *Yablo's Paradox* consists of an ω-sequence of sentences $\{S_k\}_{k\in\omega}$ where, for each $n \in \omega$:

$$S_n : (\forall k)(k > n \rightarrow False(S_k)).$$

Within L_P an equivalent construction can be obtained using infinite conjunction in place of universal quantification - the sentence names are $\{S_i\}_{i\in\omega}$ and the denotation function is given by:

$$\delta(S_i) = \wedge\{F(S_k) : k\rangle i\}.$$

We can express this in more familiar terms as:

$$\begin{aligned}
&S_1 : F(S_2) \wedge F(S_3) \wedge \cdots \wedge F(S_n) \wedge F(S_{n+1}) \wedge \cdots\\
&S_2 : F(S_3) \wedge F(S_4) \wedge \cdots \wedge F(S_n) \wedge F(S_{n+1}) \wedge \cdots\\
&S_3 : F(S_4) \wedge F(S_5) \wedge \cdots \wedge F(S_n) \wedge F(S_{n+1}) \wedge \cdots\\
&\text{etc.}
\end{aligned}$$

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[1]The class $\{S_\alpha\}_{\alpha\in A}$ may be either a set or proper class, where A is any appropriate class of indices.

[2]Intuitively, $\wedge[\{F(S_\beta)\}\beta \in B]$ is the (possibly infinitary) conjunction asserting that each S_β is false, i.e., $F(S_{\beta_1}) \wedge F(S_{\beta_2}) \wedge \cdots \wedge F(S_{\beta_i}) \wedge \cdots$ I shall use the latter notation when convenient.

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