THE BULLETIN OF SYMBOLIC LOGIC
Volume 14, Number 1, March 2008

FORCING AXIOMS, SUPERCOMPACT CARDINALS, SINGULAR CARDINAL COMBINATORICS

MATTEO VIALE

The purpose of this communication is to present some recent advances on the consequences that forcing axioms and large cardinals have on the combinatorics of singular cardinals. I will introduce a few examples of problems in singular cardinal combinatorics which can be fruitfully attacked using ideas and techniques coming from the theory of forcing axioms and then translate the results so obtained in suitable large cardinals properties.

The first example I will treat is the proof that the proper forcing axiom PFA implies the singular cardinal hypothesis SCH, this will easily lead to a new proof of Solovay's theorem that SCH holds above a strongly compact cardinal. I will also outline how some of the ideas involved in these proofs can be used as means to evaluate the "saturation" properties of models of strong forcing axioms like MM or PFA.

The second example aims to show that the transfer principle $(\aleph_{\omega+1}, \aleph_\omega) \twoheadrightarrow (\aleph_2, \aleph_1)$ fails assuming Martin's Maximum MM. Also in this case the result can be translated in a large cardinal property, however this requires a familiarity with a rather large fragment of Shelah's $\mathtt{pcf}$-theory.

Only sketchy arguments will be given, the reader is referred to the forthcoming [25] and [38] for a thorough analysis of these problems and for detailed proofs.

The singular cardinal problem. Cardinal arithmetic is a central subject in modern set theory and one of the key problems in this domain is to evaluate the gimel function $\kappa \mapsto \kappa^{\mathtt{cof}(\kappa)}$ for a singular cardinal κ. There are various reasons why this question has become so relevant. First of all it is a folklore result that the behavior of the exponential function κ^λ is completely determined by the interplay between the gimel function and the powerset function $\lambda \mapsto 2^\lambda$ restricted to the class of regular cardinals (see [15] I.5). Two standard exercises in a graduate course in set theory are to show Cantor's inequality $2^\lambda > \lambda$ for all λ and to prove that $\kappa^{\mathtt{cof}(\kappa)} > \kappa$ for all singular κ.

Received October 10, 2007.

This research has been partially supported by the Austrian Science Fund FWF project P19375-N18. The author thanks Boban Veličković and the referee for many useful comments on the redaction of this paper.

1079-8986/08/1401-0005/$2.50