MICHAEL RATHJEN. *The superjump in Martin-Löf type theory*. ***Logic Colloquium '98, Proceedings of the annual European summer meeting of the Association for Symbolic Logic, held in Prague, Czech Republic, August 9–15, 1998***, edited by Samuel R. Buss, Petr Hájek, and Pavel Pudlák, Lecture notes in logic, no. 13, Association for Symbolic Logic, Urbana, and A K Peters, Natick, Mass., 2000, pp. 363–386.

Using the concept of universes of types in Martin-Löf type theory, ML, Rathjen studies the notion of the superjump in constructive mathematics. After presenting a brief history of universes in type theory, he introduces a system of type theory MLF by means of a universe constructor. This constructor assigns to each operator F from families of sets to families of sets a universe closed under F. This gives rise to the superjump in type theory. Rathjen characterizes the system MLF mainly in terms of two systems of set theory, both based on the concept of Mahloness. The first system, KPM^r, is a system of classical admissible set theory with set foundation axiomatizing a recursively Mahlo universe. The second system is based on Aczel's constructive set theory CZF enriched by a rule (M). This rule is a reflection rule, which asserts that arbitrary true sentences are reflected on set-inaccessible sets. Additionally, Rathjen introduces a hierarchy of subsystems of $\mathsf{CZF} + (\mathsf{M})$ with n-inaccessible sets, CZF_n, exhausting the strength of $\mathsf{CZF} + (\mathsf{M})$. Moreover, he gives an interpretation of CZF_n within MLF and an interpretation of MLF within KPM^r, the latter utilizing realizability models. He also claims that all systems under discussion are of the same proof-theoretic strength.

At the end of Rathjen's paper there is a discussion about the boundaries of Martin-Löf type theory and a discussion about so-called old and new Martin-Löf type theory.

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SOLOMON FEFERMAN. *Computation on abstract data types. The extensional approach, with an application to streams*. ***Annals of pure and applied logic***, vol. 81 (1996), pp. 75–113.

To quote from the opening sentence: "This paper is a continuation of the work of Feferman [in a pair of 1992 papers] which initiated an approach through a form of *generalized recursion theory* (g.r.t.) to computation on *abstract data types* (ADTs). ... [W]e separate out the extensional part of the theory and show how it may be applied to computation on streams as an ADT. One of the main new contributions here is an explanation of how this is to be done for finite 'nonterminating' streams as well as infinite streams, and even more general partial ('gappy') streams."

For the present purpose, ADT's are simply classes of structures closed under isomorphism. A paradigm is provided by A-list structures, for an arbitrary set A. Such structures are characterised up to isomorphism relative to A (by the operations *Cons*, *Head* and *Tail*, and *nil*) as being the least structure containing *nil* and closed under *Cons*.

When one tries to construct, by analogy, the ADT of A-streams (i.e., possibly infinite sequences of elements of A), problems arise, for example in connection with the possibility of a similar characterisation up to isomorphism.

We work over many-sorted "functional structures" $\mathcal{A} = (A_0, A_1, \ldots, A_n, F_0, \ldots, F_m)$ where each F_k is a functional of type level $\leqq 2$ over the A_i's of specified arity, and A_0 is the Boolean set $\{\mathsf{tt}, \mathsf{ff}\}$. The *signature* Σ of $\mathcal{A}$ is $(n, \langle \overline{\sigma}_k, \overline{\imath}_k, j_k \rangle_{k \leqq m})$, i.e., a listing of n, m and the types of the F_k's.

We review notation and basic concepts. The basic type-one objects are *partial functions* on A, which means that one has to deal with the semantics of partially defined terms. So $t\downarrow$ means that t is defined; $t_1 = t_2$ means $t_1\downarrow \wedge t_2\downarrow \wedge t_1 = t_2$, and $t_1 \simeq t_2$ means $(t_1\downarrow \vee t_2\downarrow \Rightarrow t_1 = t_2)$. Also $f : B \xrightarrow{\sim} C$ means that f is a partial function from B to C, and $f : B \to C$ means that f is total. Let i, j, k, l range over the sort indices $1, \ldots, n$, and let $\overline{\imath}, \overline{\jmath}, \ldots$ range over finite sequences of these. For $\overline{\imath} = i_1, \ldots, i_\nu$, write $\mathrm{lh}(\overline{\imath}) = \nu$, and let $A_{\overline{\imath}} = A_{i_1}, \ldots, A_{i_\nu}$, with