looked as though the subject could be divorced from logic after all. That this is not possible was subsequently shown by Eklof and Shelah (in the third of the listed papers): The validity of Osofsky's conjecture is in fact undecidable. The fourth paper, a sequel to the third, refined and supplemented the results of the former. We start with a loose sketch of the decisive concepts of these two papers—more precision can be found in the final paragraph. The *type* of an arbitrary uniserial module U over a valuation domain R is a certain standard uniserial module associated with U (the attribute "standard" meaning "isomorphic to a subfactor of the quotient field of R, viewed as an R-module"). Inspired by Osofsky's work, Eklof and Shelah assign to any such type an invariant living in a Boolean algebra of equivalence classes of subsets of ω_1, the equivalence relation and partial order being induced by the filter of closed unbounded subsets of ω_1, via "$S_1 \cap C = S_2 \cap C$" and "$S_1 \cap C \subseteq S_2 \cap C$" respectively, for some closed unbounded C. This invariant associated with a uniserial type measures the wealth of certain ideals L in R having the property that the factor ring R/L is not complete in a type-dependent linear topology. Very roughly, one might say "the richer the supply of such ideals, the better the chance of finding non-standard uniserial R-modules of the corresponding type."

To provide meaningful detail concerning some key results of the last two papers, we need to sharpen these concepts; for simplicity, we will restrict our attention to uniserial domains R of cardinality $\aleph_1$. Given an R-subfactor J/I of the quotient field Q of R, a uniserial module U is said to be of type J/I if $J/I \cong D(u)/\operatorname{Ann}_R(u)$ for some non-zero element $u \in U$, where $\operatorname{Ann}_R(u)$ is the annihilator of u in R and $D(u)$ is the R-submodule of Q generated by the inverses r^{-1} of those elements $r \in R \setminus \{0\}$ that satisfy $u \in rU$ (this construction is independent of the choice of $u \in U \setminus \{0\}$); then U is non-standard precisely when $U \not\cong J/I$. The corresponding invariant $\Gamma(D(u)/\operatorname{Ann}_R(u))$ is as follows: If $D(u) = \bigcup_{\mu<\omega_1} r_\mu^{-1}R$ with $r_\mu \mid r_\nu$ whenever $\mu < \nu$, the value of $\Gamma(D(u)/\operatorname{Ann}_R(u))$ in the described Boolean algebra is the equivalence class of the set

$$S = \{\alpha \mid \alpha \text{ is a limit ordinal } < \omega_1 \text{ such that } R/\big(\bigcap_{\mu<\alpha} r_\mu R\big) \text{ is not complete}\}.$$

If 1 denotes the largest element of our Boolean algebra, Osofsky's conjecture takes on the form, "R has a non-standard uniserial module of type J/I precisely when $\Gamma(J/I) = 1$" (still restricted to valuation domains of cardinality $\aleph_1$). Eklof and Shelah confirm it under the negation of the continuum hypothesis. Moreover, they show that $\Gamma(J/I) = 1$ always guarantees the existence of a non-standard uniserial of type J/I. On the other hand, the converse is independent of ZFC with the generalized continuum hypothesis. As for Γ-invariants at the opposite end of the spectrum, $\Gamma(J/I) = 0$ excludes the existence of a non-standard uniserial of type J/I. As might be expected, the constructible universe again provides a "clean" outcome: For $V = L$, the existence of non-standard uniserial R-modules of type J/I is equivalent to $\Gamma(J/I) \neq 0$.

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Four papers related to the normal Moore space conjecture

PETER J. NYIKOS. *A provisional solution to the normal Moore space problem*. ***Proceedings of the American Mathematical Society***, vol. 78 (1980), pp. 429–435.

WILLIAM G. FLEISSNER. *If all normal Moore spaces are metrizable, then there is an inner model with a measurable cardinal*. ***Transactions of the American Mathematical Society***, vol. 273 (1982), pp. 365–373.