the numerous proofs of this fact also provide methods for extracting witnessing terms from classical proofs of Σ_1-formulas, either via a translation of PA into HA, or by directly giving a constructive interpretation for PA.

The paper under review introduces a new method of interpreting proofs in PA constructively, based on a new kind of formalized realizability for a Tait-style, one-sided sequent-calculus. This interpretation is quite appealing as it gives the rules of this calculus a very direct and natural computational meaning, and might therefore be of practical interest for program extraction from classical proofs.

The author also introduces a variant of the Gödel–Gentzen double-negation translation of PA into HA with respect to which his new realizability interpretation corresponds to Kreisel's modified realizability followed by the Dragalin–Friedman translation. This means that the witnessing term for a PA-proof of a Σ_1-formula is the same as the one extracted via modified realizability from the translated proof. Furthermore, he shows that the same witnesses are obtained by applying suitable cut reduction rules to the classical PA-proofs directly, which, as a side result, implies that certain strategies for eliminating cuts and extracting witnesses are insensitive to the order in which the reductions are applied.

The author continues with investigations of finitary and infinitary cut elimination procedures *á la* Gentzen and Mints. Among other results, it is shown that the witnesses obtained via these methods are, again, the same as those obtained by the methods discussed above.

It is also pointed out in the paper that there are similarities between this work and direct computational semantics for classical proofs investigated by Murthy, Parigot, and others. The reviewer would like to add that it would be interesting to clarify the precise relationship between these methods and also the relations to other methods, based, for example, on Gödel's Dialectica interpretation.

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ALBERT VISSER. *Submodels of Kripke models*. ***Archive for mathematical logic***, vol. 40 (2001), pp. 277–295.

When one tries to translate a classical result to an intuitionistic setting, one often arrives at a more complicated statement with a more complicated proof. Nonetheless, as this article by Visser illustrates, this does not necessarily imply that the result is less elegant.

The author investigates the intuitionistic analogue of the Łoś–Tarski theorem for classical predicate logic CQC which states that the formulas of first-order predicate logic that are closed under taking (classical) submodels are precisely the purely universal formulas. He arrives at the following intuitionistic analogue: the formulas of the intuitionistic predicate calculus IQC that are closed under taking submodels are precisely the semi-positive formulas. Here a submodel of a Kripke model is the result of restricting the frame of the original model. The semi-positive formulas are the formulas in which only implications of the form $(P \to A)$ occur, where P is an atomic formula. Note that both notions are superfluous in a classical setting: any class of classical models is (as a class of Kripke models) closed under submodels, and any first-order formula is classically equivalent to a semi-positive formula.

As the author points out in the introduction, the theorem and its proof are exactly as one would expect them to be. They can be seen as an extension of a similar result in intuitionistic propositional logic by Visser, van Benthem, de Jongh, and Renardel de Lavalette, which states that the (propositional) formulas preserved under subformulas are precisely the NNIL-formulas. An NNIL-formula is a formula in which only implications of the form $(p \to A)$ occur, where p is a propositional variable.

The paper closes with three other results on submodels of Kripke models: one in the context of IQC and the other two in the context of Heyting arithmetic HA. All these theorems,