

strong normalization of T by defining a new assignment $[]_0 : T \rightarrow \omega$ which decreases under one-step reduction \triangleright^1 of terms in T , i.e., $t \triangleright^1 t'$ entails $[t']_0 < [t]_0$. Moreover, $[]_0$ is an ε_0 -recursive function. As a byproduct, optimal bounds for the natural fragments T_n of T are obtained.

As the title of the paper suggests, this new analysis of Gödel's T has to be seen as a specific case study of a more general program, which has its roots in infinitary proof theory. There the general concept of *miniaturization* and *concretion* of certain (large) cardinals has long been central in the development of ordinal notation systems. Further, as has been shown by Weiermann, it is often possible to miniaturize or project down the ordinal analysis of a (strong) system in order to obtain a perspicuous proof-theoretic analysis of a corresponding weaker system. For example, he has shown in *How to characterize provably total functions by local predicativity* (*The Journal of Symbolic Logic*, vol. 61 (1996), pp. 52–69) how to pin down the standard local predicativity treatment (due to Pohlers) of the theory of one inductive definition ID_1 to a technically smooth analysis of Peano arithmetic PA. In this miniaturization, the collapsing function $D : \varepsilon_{\Omega+1} \rightarrow \Omega$ is replaced by the miniaturized collapsing function $\psi : \varepsilon_0 \rightarrow \omega$. The treatment of PA thus obtained indeed also produces optimal bounds for the fragments $I\Sigma_{n+1}$ of PA.

The treatment of Gödel's T in the paper under review can be seen as a miniaturization of Howard's analysis of bar recursion of type zero and is inspired by the local predicativity approach to pure proof theory. In particular, the definition of the assignment $[]_0$ mentioned above makes crucial use of the miniaturized collapsing function ψ . Whereas previous treatments of the subsystems T_n of T (in which the recursors have type level less than or equal to $n + 2$) used the ordinal bound ω_{n+3} (e.g. in Weiermann's paper *A proof of strongly uniform termination for Gödel's T by methods from local predicativity*, *Archive for Mathematical Logic*, vol. 36 (1997), pp. 445–460—a precursor of the article under review), the present paper yields the optimal strong normalization bound ω_{n+2} for T_n .

This is a very important paper both from the technical as well as from the conceptual point of view. In a long and sparkling introduction, the author outlines his vision of how to apply a certain kind of infinitary methods to questions of finitary mathematics, and he discusses exciting connections of his results with term rewriting theory, hierarchy theory, and computational complexity. The paper is largely self-contained and, due to its extensive motivation and conceptual discussion, it should be accessible to a wide readership.

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ULRICH KOHLENBACH. *Relative constructivity*. *The Journal of Symbolic Logic*, vol. 63 (1998), pp. 1218–1238.

In this article, the author continues his investigations of the rate of growth of functions definable in certain subsystems of analysis in finite types. In previous papers (e.g. *Mathematically strong subsystems of analysis with low rate of growth of provably recursive functionals*, BSL VII 280) the classical systems $G_n A_i^\omega$ were analyzed and proof-theoretic tools such as monotone functional interpretation (introduced by the author) were used. In the paper under review, Kohlenbach studies the intuitionistic versions $G_n A_i^\omega$ of these systems and analyzes them via a new monotone realizability interpretation. He shows that the addition of certain strong non-constructive and even classically refutable analytical and logical principles has no impact on the rate of growth of definable functions.

The main result of the paper is summarized in the following.

Main theorem. Let $E-G_n A_i^\omega$ be the system $G_n A_i^\omega$ plus extensionality axioms in all finite types, the full axiom of choice AC in all finite types, and the independence of premiss principle IP_{\neg} for negated formulas. Furthermore, let \mathcal{A} consist of the following (classically