by *proof polynomials*, which can be concatenated and applied to one another. The logic is classical, with the usual propositional variables and connectives; but whenever t is a proof polynomial and φ is a formula, $t : \varphi$ is a new proposition, intended to denote that t is a proof of φ. To interpret the S4 axiom $\Box p \rightarrow \Box\Box p$, Artemov introduces one additional operator: whenever t is a proof of φ, $!t$ is intended to denote a proof that t is a proof of φ. In other words, for each t and φ, LP has axioms of the form $t : \varphi \rightarrow !t : t : \varphi$.

Artemov shows that the system works as advertised. Any derivation in LP can be projected "forgetfully" to a derivation in S4, and, conversely, any formula that can be derived in S4 has an explicit realization that can be derived in LP. Together with Gödel's interpretation, this shows that a formula is derivable in intuitionistic logic if and only if an appropriate realization is derivable in LP. Artemov also shows how to give LP an arithmetic interpretation relative to any standard proof predicate for, say, Peano arithmetic, and he proves soundness and completeness with respect to such interpretations. This in turn yields cut-elimination theorems for suitable sequent formulations of LP.

The historical notes and references are exceptionally thorough, and this paper will serve not only as a standard reference to the various attempts to come to formal terms with the BHK interpretation, but also more generally as a useful source of information for the semantics of intuitionistic and modal logic. Artemov is admirably clear in laying out the motivations and relevant background information, and I have little to add to his exposition.

Because Artemov takes his interpretation to offer a "semantics" for intuitionistic logic, I may, perhaps, indulge in a brief reflection on this notion. I can think of three reasons that one might seek a formal semantics for a deductive system that is already in hand: (1) one might want to explicate the meaning of the deductive formalism in terms that are intuitively prior, or independently interesting; (2) one might want to have a tool for studying the deductive system itself, such as for proving independence or establishing other metamathematical properties; or (3) one may intend the semantics itself (perhaps coupled with the associated deductive system) to have applications to areas outside logic, such as mathematics, linguistics, or computer science.

Artemov's work succeeds with respect to (1). For example, the simple reflective rules of LP (and the use of a fixed-point lemma in obtaining arithmetic interpretations) clarifies the circularity needed to make sense of the S4 notion of provability. With respect to (2), it is not clear whether LP can offer anything that cannot be obtained by using, say, Kripke structures or cut elimination; in any event, this line is not pursued. With respect to (3), it is possible that LP, viewed as a term calculus, can help provide a foundational framework for reasoning about functional programming languages in which programs are equipped to construct pieces of their own code. Artemov raises the possibility of similar applications in the realm of formal verification. It will be interesting to see if such hopes are borne out.

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GAISI TAKEUTI. *Incompleteness theorems and S_2^i versus S_2^{i+1}*. ***Logic Colloquium '96, Proceedings of the colloquium held in San Sebastián, Spain, July 9–15, 1996***, edited by J. M. Larrazabal, D. Lascar, and G. Mints, Lecture notes in logic, no. 12, Springer, Berlin, Heidelberg, New York, etc., 1998, pp. 247–261.

GAISI TAKEUTI. *Gödel sentences of bounded arithmetic*. ***The journal of symbolic logic***, vol. 65 (2000), pp. 1338–1346.

"Bounded arithmetic" subsumes different theories, which in their expressive power and strength are usually included in the fragment $I\Delta_0 + \exp$ of Peano arithmetic in which induction is restricted to bounded formulas and exponentiation is a total function. In the reviewed papers, bounded arithmetic (henceforth BA) means the theory $I\Delta_0 + \Omega_1$ (Ω_1 expresses a