The foundational work of Hilbert, Weyl, and Brouwer has received a large amount of attention ever since the "Grundlagenstreit" of the 1920's. It should be safe to say that there is a general understanding of the positions of the main protagonists of the early foundational debates. Beyond a general understanding, detailed historical-foundational work is an entirely different matter and requires, besides specialized knowledge in logic and mathematics, experience in the methods of historical research. Such detailed work for the period in question is still largely to be done; for example, work on the Hilbert school of the 1920's is quite recent, and is exemplified in this volume by the essay of Wilfried Sieg.

The editors open their *Introduction* by describing proof theory as "a technical sub-field of mathematical logic." This should hardly be the attitude of most proof theorists who view their work as part of a distinctly foundational enterprise: the study of the structure of mathematical proofs. Classic examples of foundational results, in contrast to "technical" (whatever that may be), include the full mastery of the structure of proofs in first-order logic through Gentzen's sequent calculus, and the consistency of arithmetic and the determination of its proof-theoretic strength. There is any amount of subsequent work along similar lines, "technical" only insofar as that is considered a synonym for difficult.

This volume has a thematic unity that places it above most other books originating in conference papers. Its usefulness for those interested in the earlier phases of the foundational debate is further increased by a detailed index, for which the editors deserve a sincere thanks.

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SERGEI N. ARTEMOV. *Explicit provability and constructive semantics*. ***The bulletin of symbolic logic***, vol. 7 (2001), pp. 1–36.

The meaning of the intuitionistic connectives is often explained, informally, in terms of the Brouwer–Heyting–Kolmogorov (BHK) interpretation: a proof of $A \wedge B$ consists of a proof of A paired with a proof of B, a proof of $A \to B$ consists of a procedure for transforming a proof of A into a proof of B, and so on. The simplicity and intuitive appeal of this explanation suggests that there should be a formal semantics lurking underneath. But, after surveying the existing semantics for intuitionistic logic (including Kripke and Beth models, algebraic and topological semantics, realizability, and various syntactic interpretations), Artemov concludes that none of them fit the bill. He then undertakes the challenge of providing one that does.

Artemov's solution involves interpreting intuitionistic propositional logic in a logic LP of propositions and proofs. Consideration of similar interpretations will provide some useful context. Many researchers in constructive logic take the Curry–Howard (or "propositions as types") isomorphism, formally represented by deductive type theories like Martin-Löf's, to offer a formal explication of the BHK interpretation. But Artemov objects that this does not go far enough: such type theories *name* the associated proof objects, but do not supply the linguistic resources to reason about them as proofs per se. As a logic of provability, Gödel's modal logic S4 fares better in this respect: for any proposition, p, one can also express the proposition, $\Box p$, that p is provable. Indeed, Gödel's 1933 interpretation of Heyting's propositional calculus in S4 shows that intuitionistic logic can be understood in terms of such a notion of provability. But Gödel also pointed out that one cannot interpret the S4 box operator in terms of a proof predicate for an arithmetic deductive system, since, under such an interpretation, the S4 theorem $\Box(\Box\bot \to \bot)$ asserts that the system can prove its own consistency. So the challenge amounts to showing how one can understand the S4 notion of "provability" in terms of actual "proofs."

Artemov's system LP draws from both the modal and type-theoretic approaches, and, given the constraints he has set up, provides a natural solution. Proof objects are represented