

Chapter 4 works out the axioms of basic set theory. It turns out that the basic existence axioms of set theory, other than the axiom of infinity, generate finite (determinate) sets—including the axioms of replacement and power set, which in the latter case may seem surprising to those who doubt the “determinateness” of the power set operation. So basic set theory is ZFC without the axiom of infinity.

Part Three is a careful development of the elements of Cantorian set theory from the axiom of infinity. At the end of Chapter 5, the author argues, contra Zermelo and Gödel, that there are no grounds for the existence of cardinal numbers beyond those implied by ZFC. He considers the case of Mahlo cardinals or more precisely of regular closure cardinals  $\mu$  for a global ordinal function  $\sigma$  ( $\alpha < \mu$  implies  $\sigma(\alpha) < \mu$ ). (If such a  $\mu$  exists for each normal function  $\sigma$ , then the totality of all ordinals is Mahlo.) But the argument for the existence of  $\mu$  that he rejects is not the argument that Gödel himself found convincing. The argument that Gödel *did* find convincing is a straightforward generalization of what seems like the most convincing argument for the axiom of replacement: if for no ordinal  $\alpha$  in the initial segment  $X$  of ordinals does  $R(\alpha)$  have a certain property and  $R(X)$  does have the property, then  $X$  should be admitted as an ordinal: it is “finite” in the author’s sense because it is determined by the condition of being the least ordinal with the property. The properties in question are those of satisfying some formula  $\varphi(A)$ , where  $A$  is a class. (From the point of view of  $R(\alpha)$ ,  $A$  is  $A \cap R(\alpha)$ .) Since  $A$  is a species and so infinite on the author’s view, he would perhaps want to reject this kind of argument. But in the case of the axiom of replacement, where, say,  $\varphi(A)$  expresses that  $A$  is a function defined on some  $\alpha \in X$  and whose values are in  $X$ , it seems to me that this is the best argument there is for the “finiteness” of  $X$ , of why we should, on the author’s grounds, accept it as a set.

Part Four develops Euclidean set theory, based on the axiom that every set is Dedekind finite. In Chapter 10, there is an attractive development of a Euclidean theory of simply infinite systems. A simply infinite system based on the global function  $\sigma$  and the object  $a$  is a species of all linear orderings with first member  $a$  and in which each member other than the first is obtained from its predecessor by means of  $\sigma$ .  $\sigma$  and  $a$  determine a simply infinite system iff for no linear ordering  $r$  in it is the value of  $\sigma$  applied to the last element of  $r$  in the field of  $r$ . One should note that, in Quine’s *Set theory and its logic* (JSL XXXVII 768) (which the author justly criticizes for its common-sense-is-bankrupt approach to set theory), there is another quite elegant treatment of simply infinite systems that does not depend on the axiom of infinity (or on its negation). On Quine’s conception the elements of the system are the  $x$  that satisfy the condition:  $\forall z[x \in z \wedge \forall y \in z(\sigma^*(y) \subseteq z) \rightarrow a \in \sigma]$ , where  $\sigma^*(y)$  is the set of pre-images of  $y$ . Quine treats the special case of  $\sigma(x) = x \cup \{x\}$  and  $a = 0$ , but the treatment generalizes.

The book under review is a very lively book, filled with striking theses, for the most part clearly formulated and argued for in detail. I found, reading it, that my head was continually shaking, sometimes positively and sometimes negatively. I confess that on philosophical matters, it probably shook more often no; but, I always felt engaged and—such is the nature of philosophy—it would not surprise me if other readers found themselves more in agreement with the author.

W. W. TAIT

Department of Philosophy, University of Chicago, Chicago, IL 60637, USA.  
wwtx@uchicago.edu.

OLIVER ABERTH. *Computable calculus*. Academic Press, San Diego, London, etc., 2001, xiii + 192 pp. + CD-ROM.

The title of this interesting and unusual book refers to calculus based on interval arithmetic and carried through by using a model of computation equivalent to a Turing machine. As anyone familiar with computability theory will realise, there are many differences between