

YOSHIHIRO ABE. *Weakly normal filters and the closed unbounded filter on  $P_\kappa\lambda$* . *Proceedings of the American Mathematical Society*, vol. 104 (1988), pp. 1226–1234.

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YOSHIHIRO ABE and MASAHIRO SHIOYA. *Regularity of ultrafilters and fixed points of elementary embeddings*. *Tsukuba journal of mathematics*, vol. 22 (1998), pp. 31–37.

Throughout this review,  $\kappa$  will denote a regular uncountable cardinal and  $\lambda$  a cardinal  $> \kappa$ .  $P_\kappa\lambda$  is the collection of all subsets of  $\lambda$  of size  $< \kappa$ . As a convenient convention, the phrase “ideal on  $P_\kappa\lambda$ ” will mean “proper,  $\kappa$ -complete, fine ideal on  $P_\kappa\lambda$ .”

Since the formulation of the concept of supercompactness by Reinhardt and Solovay in the late sixties, much work has gone into an investigation of the properties of individual ideals on  $P_\kappa\lambda$ . Nevertheless, as pointed out by Kanamori in *The higher infinite*, p. 351 (JSL LXI 334), for ideals on  $P_\kappa\lambda$  “a full structure theory has yet to be worked out.” The seven papers under review represent a selection of Abe’s results in this area.

Suppose  $\kappa$  is  $\lambda$ -supercompact. Menas (*A combinatorial property of  $p_\kappa\lambda$* , JSL LVI 1098) showed that if  $\lambda$  is a strong limit cardinal of cofinality  $< \kappa$ , then  $P_\kappa\lambda$  bears two distinct isomorphic prime ideals extending the non-stationary ideal  $\text{NS}_{\kappa\lambda}$ . In the first paper the author proves that the conclusion still holds if  $\lambda$  is inaccessible.

The remainder of the paper and the next two papers are concerned with Kanamori ideals. An ideal  $J$  on  $P_\kappa\lambda$  is *Kanamori* if for every regressive  $f : P_\kappa\lambda \rightarrow \lambda$ , there exist  $B \in J^*$  and  $\gamma < \lambda$  such that  $f(b) \leq \gamma$  for every  $b \in B$ . (The corresponding property for ideals on  $\kappa$  was studied by Kanamori in *Weakly normal filters and irregular ultrafilters*, *Transactions of the American Mathematical Society*, vol. 220 (1976), pp. 393–399, whence the name).  $J$  is *weakly normal* if given  $A \in J^+$  and a regressive  $f : P_\kappa\lambda \rightarrow \lambda$ , there exist  $B \in J^+ \cap P(A)$  and  $\gamma < \lambda$  such that  $f(b) \leq \gamma$  for every  $b \in B$ . The two notions appear in the literature under various names. Abe himself varies in his terminology. By a “weakly normal ideal” he means what I call a Kanamori ideal in the second paper and what I call a weakly normal ideal in the fifth one. Note that the concept of weak normality is useless if  $\text{cf}(\lambda) < \kappa$ , since then every ideal on  $P_\kappa\lambda$  is weakly normal. Also, the two notions coincide if  $J$  is prime:  $J$  is Kanamori iff it is weakly normal. More generally, as shown by Abe in the third paper, an ideal  $J$  on  $P_\kappa\lambda$  is Kanamori iff it is weakly normal and every disjoint family of sets in  $J^+$  has size  $< \text{cf}(\lambda)$ .

In the first paper only *prime* Kanamori ideals are considered. There the author makes the simple but crucial observation that every prime ideal on  $P_\kappa\lambda$  has a prime Kanamori ideal below it in the RK-ordering. Thus there always exists a prime Kanamori ideal on  $P_\kappa\lambda$  in case  $\kappa$  is  $\lambda$ -compact. It is also shown that if  $\kappa$  is a measurable limit of supercompact cardinals and  $\lambda$  is regular, then there is a prime Kanamori ideal  $J$  on  $P_\kappa\lambda$  that is RK-minimal but does not extend  $\text{NS}_{\kappa\lambda}$  (in fact  $\{a \in P_\kappa\lambda : a \cap \kappa \in \kappa\} \in J$ ).

The second paper is concerned with existence of Kanamori ideals. Let  $K_{\kappa\lambda}$  assert the existence of a Kanamori ideal on  $P_\kappa\lambda$ . Abe’s first observation is that  $K_{\kappa\lambda}$  implies that  $\kappa$  is a limit cardinal. He goes on to establish that if  $K_{\kappa\lambda}$  holds and either  $2^{<\text{cf}(\lambda)} < \kappa$  or  $\text{cf}(\lambda) = \kappa$ ,  $\kappa$  is inaccessible, and there is no Suslin  $\kappa$ -tree, then  $\kappa$  is  $\lambda$ -compact. Finally, he shows that these results are sharp by establishing the consistency of the following three statements (relative