

The most important is the solution of the Moore–Mrówka problem whether countably tight compact spaces are sequential. Balogh proves that the answer *yes* is consistent. Countable tightness and sequentiality are among the most important generalizations of the property of having a countable base at each point, and play an important role in the theory of non-metrizable function spaces. A space has countable tightness if the fact that a point belongs to the closure of a subspace  $A$  means that it belongs to the closure of some countable subset of  $A$ . A space is sequential if for being a closed subspace it is enough to be closed under limits of convergent sequences. (Remember that such a nice classical compact space as  $\beta N$  does not have any non-trivial convergent sequences; it is neither countably tight nor sequential.) Several other important consequences are obtained, for example, a countably compact manifold is metrizable if and only if it does not contain a copy of  $\omega_1$ .

The complementary constructions of countably tight compact spaces that are not sequential or even have no converging sequences are due to Ostaszewski and Fedorchuk respectively. They belong to the earlier period in the history of set-theoretic topology and were obtained under the assumption of  $\diamond$ .

The link between Todorćević’s original method and Balogh’s proof was later analyzed by Alan Dow in *An introduction to applications of elementary submodels to topology* (BSL VII 537), where one more fruit was harvested: quoting the above paper, “just something that Fremlin, Nyikos and Balogh ‘missed’ ” is that PFA implies that each countably tight compact space has a point with a countable neighborhood base. Balogh’s methods also are related to research of other mathematicians most of them are coauthors of the paper by Z. Balogh, A. Dow, D. Fremlin, and P. Nyikos, *Countable tightness and proper forcing*, *Bulletin of the American Mathematical Society*, n.s. vol. 19 (1988), pp. 295–298. In this paper the authors also note that the assumption of the existence of large cardinals used in the second paper under review is not necessary to obtain its results.

The fact that the Moore–Mrówka problem is undecidable nowadays belongs to the canons of independence in mathematics. Understanding the results of the papers is “a must” for anyone seriously interested in the use of forcing in topology. Two famous open problems intimately related to the results of the papers under review are probably waiting for presently unavailable forcing methods: Is it consistent that there are no L-spaces? Is it consistent that if  $X$  is compact then it contains either a copy of  $\beta N$  or a copy of a convergent sequence (Efimov)? Therefore the papers or the study of the results of the papers (now available in other sources mentioned above) could also be suggested to all logicians and set-theorists who are developing new forcing methods.

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MAXIM R. BURKE and MENACHEM MAGIDOR. *Shelah’s pcf theory and its applications*. *Annals of pure and applied logic*, vol. 50 (1990), pp. 207–254.

The paper under review was published before Shelah’s book on pcf theory (*Cardinal arithmetic*, JSL LXII 1035) appeared, with the authors’ expressed “hope that what we do present will serve as an accessible introduction to the original papers” (p. 208). In fact the Burke–Magidor paper continued to be a frequently quoted source for pcf theory in the years after the appearance of *Cardinal arithmetic*, because Shelah’s style of writing made his book difficult to read for many.

This paper presents the fundamental theorems of pcf theory: the  $\lambda$ -directedness of  $(\prod A/J_{<\lambda})$ , the *convexity theorem* (that pcf of a small interval of regular cardinals is an interval of regular cardinals), the existence of pcf generators, and localization. Applications of pcf theory are presented also. Within just a few pages from the beginning, the existence of a Jónsson algebra on  $\aleph_{\omega+1}$  is proved. Results in the partition calculus are also presented.