Z. Szentmiklóssy. *S-spaces and L-spaces under Martin's axiom*. ***Topology***, Volume II, edited by A. Császár, Colloquia mathematica Societatis János Bolyai, no. 23, János Bolyai Mathematical Society, Budapest, and North-Holland Publishing Company, Amsterdam, Oxford, and New York, 1980, pp. 1139–1145.

Zoltán Balogh. *On compact Hausdorff spaces of countable tightness*. ***Proceedings of the American Mathematical Society***, vol. 105 (1989), pp. 755–764.

The papers under review fall under applications of set-theoretic methods in topology. In both cases the logical component of these methods is essential; in the case of the first paper it is Martin's axiom and in the case of the second paper under review it is the proper forcing axiom. One can identify two flourishing periods in the development of consistency results in set-theoretic topology, related to applications of the two above-mentioned axioms respectively. The papers can be considered as some of the leading representatives of these periods. In the meantime, extensive and excellent literature has appeared that deals with the results of the papers.

The main result of the paper of Z. Szentmiklóssy is that under the assumption of MA $+ \neg$ CH there are no compact S-spaces. MA stands for Martin's axiom which is an independent (in the logical sense) set-theoretic statement about partial orders that satisfy the countable chain condition (c.c.c.). It can be used in a variety of ways in mathematical constructions that can be approximated by a partial order. CH stands for the continuum hypothesis. S-spaces are hereditarily separable but not hereditarily Lindelöf regular spaces and L-spaces are hereditarily Lindelöf but not hereditarily separable regular spaces. These two classes of topological spaces are pretty natural, since separability and the Lindelöf property are equivalent in the class of metrizable spaces. The first examples of Hausdorff S-spaces and L-spaces, without requiring regularity, were obtained without any special set-theoretic assumptions by Wacław Sierpiński in *Sur l'équivalence de trois propriétés des ensembles abstraits*, ***Fundamenta mathematicae***, vol. 2 (1921), pp. 179–188. Nevertheless, regular (and first countable and compact) examples were obtained only under various additional set-theoretic assumptions, first by assuming CH in A. Hajnal and I. Juhász, *On hereditarily α-Lindelöf and α-separable spaces, II*, ***Fundamenta mathematicae***, vol. 81 (1974), pp. 148–158. Thus, the main result of the paper, that the usual axioms of set theory ZFC are not sufficient to construct a compact S-space, is very interesting.

In fact, the author proves stronger results which are based on a lemma in ZFC which has become known as Szentmiklóssy's lemma. (See J. Roitman, *Basic S and L*, JSL LII 1044. Roitman devotes almost the entire Section 6 to the results of the paper under review.) The arguments of the paper were later strengthened and dualized. (See for example U. Abraham and S. Todorcevic, *Martin's axiom and first-countable S- and L-spaces*, JSL LII 1044.)

In the early eighties, S. Todorcevic pioneered the use of proper forcings (a notion due to S. Shelah) to obtain undecidability of topological problems. Actually, he introduced a general approach, called *models as side conditions*, which describes how to redesign a forcing notion (a partial order used to obtain a generic extension of a model of ZFC) that fails to be c.c.c. (and thus MA cannot be applied) and obtain a proper forcing notion that may do the job of producing a consistency proof. Using this approach, among others, he solved a long-standing open problem strengthening Szentmiklóssy's result: One cannot construct even a regular S-space without special set-theoretic assumptions; the proper forcing axiom implies that there are no S-spaces. The present state of the art on S- and L-spaces, including the method of models as side conditions, is described by Todorcevic in ***Partition problems in topology*** (JSL LVI 1488). This book gives the combinatorial essence of Szentmiklóssy's result in Theorem 7.10 (see also interesting historical remarks about Szentmiklóssy's result on page 65).

Balogh's paper (the second under review) is a daring adaptation of Todorcevic's method in the context of countably compact spaces, employing also Fremlin's and Nyikos's ideas. The consequences are of historical importance.