Beklemishev puts also a strong emphasis on the base theories used. These careful distinctions help to clarify the relationship of the different axioms and rules: If n iterations of a rule correspond to n nested applications of another rule, this indicates a close relation between the two rules. Also their relationship is stronger if the equivalence obtains over a great variety of base theories. All theories considered are formulated in arithmetical languages.

If the base theory is fixed, then one can formulate reflection principles by explicitly producing a suitable provability predicate. But there is no canonical representation of arbitrary axiom sets and thus no canonical provability predicate for any arbitrarily given theory. For theories with finite axiom sets, however, there obviously are canonical provability predicates (see Feferman's *Arithmetization of metamathematics in a general setting*, JSL XXXI 269). Thus general considerations without fixed theories usually proceed in terms of reflection principles for theories with finite axiom sets. Beklemishev's paper is no exception.

His analysis of Π_n-IR is as follows. If $n \geqq 1$, T is a theory containing EA (or in the case $n = 1$ the theory $I\Delta_0 + \text{SUPEXP}$ because $[T, \Pi_n\text{-IR}]$ must prove the formalized cut-elimination theorem), then $[T, \Pi_n\text{-IR}]$ is equivalent to T together with the uniform reflection principle

$$(1) \qquad \forall x(\text{Bew}_{T_0}(\ulcorner \varphi(\dot{x}) \urcorner) \rightarrow \varphi(x))$$

for all finite Π_{n+1} axiomatized theories $T_0 \subseteq T$.

Although Σ_n-IR is known to be equivalent to Π_{n+1}-IR over EA, the above result cannot be easily transformed into an analysis of Σ_n-IR because T with the uniterated rules, i.e., $[T, \Sigma_n\text{-IR}]$ and $[T, \Pi_{n+1}\text{-IR}]$, may be different theories. Thus the following result cannot be obtained directly. If T contains $I\Sigma_n$, then $[T, \Sigma_{n+1}\text{-IR}]$ is equivalent to T together with the uniform reflection principle (1) for all finite Π_{n+2} axiomatized theories $T_0 \subseteq T$. Thus at least for Π_{n+2} axiomatized theories containing $I\Sigma_n$, $[T, \Sigma_{n+1}\text{-IR}]$ and $[T, \Pi_{n+2}\text{-IR}]$ are the same theories.

These results imply the following characterization of the induction rules in terms of iterated reflection principles: EA plus Σ_n-IR (or Π_{n+1}-IR) is equivalent to the ω-times iterated reflection principle for Σ_n formulas (starting from EA).

Beklemishev's paper is quite readable and is accessible even to readers not familiar with this area of proof theory. He introduces the basic notions carefully and reviews some well-known results in order to set the stage for his own work. The paper also contains some new proofs of old results and thereby provides new insights. The proofs are worked out in detail.

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URI ABRAHAM, MATATYAHU RUBIN, and SAHARON SHELAH. *On the consistency of some partition theorems for continuous colorings, and the structure of $\aleph_1$-dense real order types*. ***Annals of pure and applied logic***, vol. 29 (1985), pp. 123–206.

There may have been a time when it seemed that any problem related to the countable chain condition (ccc) would eventually be solved by Martin's axiom. If there was, it surely ended with the appearance of the paper under review. As the authors point out in their introduction, much of the impetus for the direction of research reported in this paper derived from the following result of Baumgartner generalizing Cantor's classic theorem that every two dense linear orders are isomorphic: It is consistent that any two $\aleph_1$-dense subsets of $\mathbb{R}$ are order-isomorphic. (A set of reals is said to be $\aleph_1$-dense provided that its intersection with each rational interval has size $\aleph_1$.) Since Baumgartner's argument relied on a finite-support iteration of ccc partial orders, as well as a great deal of cleverness, there was interest in seeing whether at least the cleverness could be dispensed with and the result obtained as a direct consequence of Martin's axiom. This,