Computer Science, June 26–29, 1995, San Diego, California, IEEE Computer Society Press, 1995, pp. 496–504) proved the surprising result that linear logic with weakening was decidable. Lafont has given another proof of it (*The finite model property for various fragments of linear logic*, ***The journal of symbolic logic***, vol. 62 (1997), pp. 1202–1208).

The present paper investigates the complexity of linear logic with weakening, LLW, and gives the best results so far. The author proves that linear logic with weakening is exponential-space hard.

The method in the paper is to give a problem that is exponential-space complete and that can be simulated in propositional linear logic with weakening. This is done by going through the following systems: TUR, ESC, CSTS, LLW^-, LLW. Let us look at these in turn.

The starting point is a problem TUR which is exponential-space complete. This is the halting problem for Turing machines space-bounded by $2^{2^{O(x)}}$. Then this is simulated by ESC, the halting problem for 3-counter machines with similar space bound. All this is of course standard and is simply referred to in the paper. The next step is perhaps the crucial one. CSTS stands for commutative, contractive, semi-Thue systems, which are semi-Thue systems where we have both commutativity and contraction rules $(xy \rightarrow x)$. For the simulation of 3-counter machines in CSTS we have the problem of representing tests of 0. In the usual representation we can represent only positive information—that a counter contains something. This is similar to the key problem in proving the undecidability of linear logic. As there, we use a representation where the only successful derivations are those where the tests of 0 are all right. Then the derivations in CSTS are simulated by cut-free derivations in linear logic with weakening, LLW^-. The last step is to give a cut-elimination result for LLW.

As a side note we observe that we need a slightly strengthened cut-free calculus. We need a sequent calculus using some extra axioms and admitting cut elimination. This was also needed in proving the undecidability of linear logic. This is a common situation and has recently been cleared up by the work of Sara Negri and Jan von Plato (***Structural proof theory***, Cambridge University Press, 2001).

The author mentions some of his work in giving doubly-exponential space as an upper bound for the complexity of LLW. The decidability proof in Kopylov's paper cited gives a bound primitive recursive in the Ackermann function. The author mentions that this can be refined considerably, to give a doubly-exponentially space bound, but he has since withdrawn that claim.

From this paper we know more about the complexity of linear logics. It would of course be nice to get an exact bound, but so far we do not know how that should be done.

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RICHARD L. EPSTEIN and WALTER A. CARNIELLI. ***Computability. Computable functions, logic, and the foundations of mathematics.*** The Wadsworth & Brooks/Cole mathematics series. Wadsworth & Brooks/Cole Advanced Books & Software, Pacific Grove, Calif., 1989, xvii + 297 pp.

RICHARD L. EPSTEIN and WALTER A. CARNIELLI. ***Computability. Computable functions, logic, and the foundations of mathematics.*** Second edition of the preceding. Wadsworth, Belmont, Calif., etc., 2000, xii + 299 + 38 pp.

RICHARD L. EPSTEIN. *Computability and undecidability—a timeline*. Therein, pp. 1–38.

The main purpose of this book is to give an introductory presentation of the theory of computable functions (of the natural numbers into the natural numbers) and to develop the fundamental aspects of logic needed to prove incompleteness, undecidability, and non-provability of consistency of some systems of arithmetic. This is a textbook appropriate for