

R. DOUGHERTY and A. S. KECHRIS. *Hausdorff measures and sets of uniqueness for trigonometric series*. *Proceedings of the American Mathematical Society*, vol. 105 (1989), pp. 894–897.

ALEXANDER S. KECHRIS and ALAIN LOUVEAU. *Covering theorems for uniqueness and extended uniqueness sets*. *Colloquium mathematicum*, vol. 59 (1990), pp. 63–79.

ALEXANDER S. KECHRIS. *Hereditary properties of the class of closed sets of uniqueness for trigonometric series*. *Israel journal of mathematics*, vol. 73 (1991), pp. 189–198.

A. S. KECHRIS and A. LOUVEAU. *Descriptive set theory and harmonic analysis*. *The journal of symbolic logic*, vol. 57 (1992), pp. 413–441.

It is a difficult (and pointless) task to draw the boundary between logic and the rest of mathematics. Part of the difficulty is due to the field of descriptive set theory. This field, which is about a hundred years old, has its origins in analysis. The first descriptive set theorists were analysts, and in its early days, the field was a subfield of analysis. The name of the field derives from this era, when an analyst, Lusin, divided the theory of real functions into two parts, which he called the *metric theory* and the *descriptive theory*. Later, the field became a subfield of topology; indeed, the canonical reference to the field was Kuratowski's book, entitled *Topology* (Volume I, PWN and Academic Press, 1966; see JSL IV 136(17)). In the last third of the twentieth century, most descriptive set theorists have been logicians, and the field is now usually considered to be a subfield of mathematical logic. At the beginning of this third era, logicians studied descriptive set theory for its own sake, or for its connections with other parts of logic. That changed in the mid-eighties, when logicians began looking at connections between various branches of analysis and descriptive set theory (including “logical” aspects of descriptive set theory, such as recursion-theoretic concepts and strong set-theoretic axioms).

Descriptive set theory not only returned to its roots in analysis, but returned to the branch of analysis in which the theory of sets began: trigonometric series. Some of the most important work done on connections between descriptive set theory and analysis involves certain types of exceptional sets, that is, notions of smallness, which occur in the study of these series. Two types of exceptional sets are the *sets of uniqueness*, or U -sets, and the *sets of extended uniqueness*, or U_0 -sets. The sets under consideration are subsets of the circle, \mathbb{T} . A set E is an M -set if there is a non-zero trigonometric series that converges to 0 outside E , and is a U -set otherwise. A set E is an M_0 -set if E supports a non-zero measure μ with Fourier coefficients $\hat{\mu}(n)$ converging to 0 as $|n| \rightarrow \infty$, and is a U_0 -set otherwise. All M_0 -sets are M -sets, since $\sum \hat{\mu}(n)e^{inx}$ converges to 0 off the set. Most of the theory concerns closed sets. The closed U - and U_0 -sets form two pointsets in the Polish space of compact subsets of \mathbb{T} , and the connection between descriptive set theory and exceptional sets in harmonic analysis begins with the following theorem of Kaufman and Solovay: Both pointsets are true Π_1^1 . Those with a serious interest in this topic should read the 1987 book, *Descriptive set theory and the structure of sets of uniqueness*, by A. S. Kechris and A. Louveau (JSL LVI 344).

All four papers under review were written after the appearance of that book. The first three are technical mathematical papers, written for a reader who is reasonably familiar with that book's contents. These three papers will be discussed individually, below. The fourth paper is different. It is a survey/expository paper, a written version of a short course given by Kechris and Louveau at the 1989 European summer meeting of the Association for Symbolic Logic. There is a substantial overlap between this paper and the Kechris–Louveau book. In addition, this paper discusses work done after the appearance of the book, including results in the first three papers under review. This paper is an excellent survey, clearly written, comprehensive and up to date (in 1992); but in contrast to the book, complete proofs are not given.