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CLASSIFICATION FROM A COMPUTABLE VIEWPOINT

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§1. Introduction. Classification is an important goal in many branches of mathematics. The idea is to describe the members of some class of mathematical objects, up to isomorphism or other important equivalence, in terms of relatively simple invariants. Where this is impossible, it is useful to have concrete results saying so. In model theory and descriptive set theory, there is a large body of work showing that certain classes of mathematical structures admit classification while others do not. In the present paper, we describe some recent work on classification in computable structure theory.

Section 1 gives some background from model theory and descriptive set theory. From model theory, we give sample structure and non-structure theorems for classes that include structures of arbitrary cardinality. We also describe the notion of Scott rank, which is useful in the more restricted setting of countable structures. From descriptive set theory, we describe the basic Polish space of structures for a fixed countable language with fixed countable universe. We give sample structure and non-structure theorems based on the complexity of the isomorphism relation, and on Borel embeddings.

Section 2 gives some background on computable structures. We describe three approaches to classification for these structures. The approaches are all equivalent. However, one approach, which involves calculating the complexity of the isomorphism relation, has turned out to be more productive than the others. Section 3 describes results on the isomorphism relation for a number of mathematically interesting classes—various kinds of groups and fields. In Section 4, we consider a setting similar to that in descriptive set theory. We describe an effective analogue of Borel embedding which allows us to make distinctions even among classes of finite structures. Section 5 gives results on computable structures of high Scott rank. Some of these results make use of computable embeddings. Finally, in Section 6, we mention some open problems and possible directions for future work.

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