The Bulletin of Symbolic Logic
Volume 11, Number 3, Sept. 2005

MATHEMATICAL EXISTENCE

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Despite some discomfort with this grandly philosophical topic, I do in fact hope to address a venerable pair of philosophical chestnuts: mathematical truth and existence. My plan is to set out three possible stands on these issues, for an exercise in compare and contrast.[1] A word of warning, though, to philosophical purists (and perhaps of comfort to more mathematical readers): I will explore these philosophical positions with an eye to their interconnections with some concrete issues of set theoretic method.

Let me begin with a brief look at what to count as 'philosophy'. To some extent, this is a matter of usage, and mathematicians sometimes classify as 'philosophical' any considerations other than outright proofs.[2] So, for example, discussions of the propriety of particular mathematical methods would fall under this heading: should we prefer analytic or synthetic approaches in geometry?[3] Should elliptic functions be treated in terms of explicit representations (as in Weierstrass) or geometrically (as in Riemann)?[4] Should we allow impredicative definitions?[5] Should we restrict ourselves to a logic without bivalence or the law of the excluded middle?[6] Also included in this category would be the trains of thought that shaped our central concepts:

Received November 18, 2004; accepted January 25, 2005.

My thanks to John Burgess, Peter Koellner, Mary Leng, John Steel, and William Demopoulos; to audiences at the 2004 Annual Meeting of the Association for Symbolic Logic (at CMU) and the 2004 Chapel Hill Colloquium; and to BSL editor Akihiro Kanamori and an anonymous referee for helpful comments on earlier versions of this paper.

[1]I do not mean to suggest these are the only possibilities; my goal is to identify some landmarks.

[2]There is a hint of this in Gödel [1964], when he describes the undecidability of the continuum hypothesis as 'a precise formulation of the ... conjecture ... that the difficulties of the problem are probably not purely mathematical' (p. 259). Of course, this was no off-hand remark; Gödel had highly developed philosophical views (see, e.g., van Atten and Kennedy [2003]).

[3]See, for example, the 19th century debate in which supporters of synthetic methods hoped 'to free geometry from the hieroglyphics of analysis' (Carnot) and the 'clatter of the coordinate mill' (Study) (Kline [1972], p. 835).

[4]See Tappenden [200?] for discussion.

[5]As predicativists do not (e.g., see Feferman [1988]).

[6]As various constructivists propose (e.g., see Bridges [2003]).

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1079-8986/05/1103-0002/$3.60