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BARWISE: ABSTRACT MODEL THEORY AND GENERALIZED QUANTIFIERS

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§1. Introduction. After the pioneering work of Mostowski [29] and Lindström [23] it was Jon Barwise's papers [2] and [3] that brought abstract model theory and generalized quantifiers to the attention of logicians in the early seventies. These papers were greeted with enthusiasm at the prospect that model theory could be developed by introducing a multitude of extensions of first order logic, and by proving abstract results about relationships holding between properties of these logics. Examples of such properties are

κ-compactness. *Any set of sentences of cardinality $\leq \kappa$, every finite subset of which has a model, has itself a model.*

Löwenheim-Skolem Theorem down to κ. *If a sentence of the logic has a model, it has a model of cardinality at most κ.*

Interpolation Property. *If ϕ and ψ are sentences such that $\models \phi \to \psi$, then there is θ such that $\models \phi \to \theta$, $\models \theta \to \psi$ and the vocabulary of θ is the intersection of the vocabularies of ϕ and ψ.*

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