THE BULLETIN OF SYMBOLIC LOGIC
Volume 9, Number 2, June 2003

MODEL THEORY: GEOMETRICAL AND SET-THEORETIC ASPECTS AND PROSPECTS

ANGUS MACINTYRE

In Memory of Maurice Boffa

§1. Introduction. I see model theory as becoming increasingly detached from set theory, and the Tarskian notion of set-theoretic model being no longer central to model theory. In much of modern mathematics, the set-theoretic component is of minor interest, and basic notions are geometric or category-theoretic. In algebraic geometry, schemes or algebraic spaces are the basic notions, with the older "sets of points in affine or projective space" no more than restrictive special cases. The basic notions may be given sheaf-theoretically, or functorially. To understand in depth the historically important affine cases, one does best to work with more general schemes. The resulting relativization and "transfer of structure" is incomparably more flexible and powerful than anything yet known in "set-theoretic model theory".

It seems to me now uncontroversial to see the fine structure of definitions as becoming the central concern of model theory, to the extent that one can easily imagine the subject being called "Definability Theory" in the near future.

§2. Tarskian beginnings.

2.1. Tarski's set-theoretic foundational formulations are still favoured by the majority of model-theorists, and evolution towards a more suggestive language has been perplexingly slow. None of the main texts uses in any nontrivial way the language of category theory, far less sheaf theory or topos theory. Given that the most notable interactions of model theory with geometry are in areas of geometry where the language of sheaves is almost indispensable (to the geometers), this is a curious situation, and I find it hard to imagine that it will not change soon, and rapidly.

2.2. In Tarski's foundational scheme, everything happens within a (the?) universe of set theory. All the entities of the subject (semantic and syntactic) are sets (or, now and then, classes). **Structures** are defined as (rather

Received January 15, 2002; accepted September 29, 2002.

© 2003, Association for Symbolic Logic
1079-8986/03/0902-0005/$2.60