

## FROM FOUNDATIONS TO LUDICS

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*Ludics* [1] is a novel approach to logic—especially proof-theory. The present introduction emphasises foundational issues.

**§1. All Quiet on the Western Front.** For ages, not a single *disturbing* idea in the area of “foundations”: the discussion is sort of ossified—as if everything had been said, as if all notions had taken their definite place, in a big cemetery of ideas. One can still refresh the flowers or regild the stone, e.g., prove technicalities, sometimes non-trivial; but the real debate is still: this paper begins with an *autopsy*, the autopsy of the foundational project.

**1.1. Realism.** Up to say 1900, the realist/dualist approach to science was dominant; during the last century some domains like physics evolved so as to become completely anti-realist; but this evolution hardly concerned logic.

**1.1.1. Hilbert’s legacy.** By the turn of the XX<sup>th</sup> century mathematics was jeopardised by paradoxes, the most famous of them being due to Russell. Hilbert’s reaction was to focus on *consistency*. But the reduction of paradoxes—and therefore of foundations—to solely the *antinomies* is highly questionable:<sup>1</sup> indeed, the typical paradoxical artifacts are *secret sharers*, objects satisfying the formal definitions but far astray from the intended meaning, typically the Peano “curve” which contradicts our perception of *dimension*. Fortunately, topology has been able to show that dimension  $m$  is not the same as dimension  $n \dots$  but just for a second, forget this and imagine *consistent* mathematics in which balls in any dimension are homeomorphic: what a disaster! This exclusive focus on consistency—not to speak of the strategic failure of the Programme—should explain why logic, especially *foundations* lost contact with other sciences during last century.

Indeed Hilbert’s Programme is not quite realistic, it is *procedural*, see 1.3 below: Hilbert tried to avoid as much as possible the external *reality*. This was wise; but he made several mistakes, both technical and methodological:

- We shall see that Hilbert’s Programme relies on a duality between proofs of  $A$  and proofs of  $\neg A$ . But the dualising object, the *pivot* of this

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<sup>1</sup> $\delta\acute{o}\xi\alpha$ : dogma, opinion, intuition . . . : a *paradox* need not be a formal contradiction.