

GÖDEL, TARSKI, CHURCH, AND THE LIAR

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The fact that Gödel's famous incompleteness theorem and the archetype of all logical paradoxes, that of the Liar, are related closely is, of course, not only well known, but is a part of the common knowledge of the community of logicians. Indeed, almost every more or less formal treatment of the theorem makes a reference to this connection. Gödel himself remarked in the paper announcing his celebrated result (cf. [7]):

The analogy between this result and Richard's antinomy leaps to the eye; there is also a close relationship with the 'liar' antinomy, since . . . we are . . . confronted with a proposition which asserts its own unprovability.

In the light of the fact that the existence of this connection is commonplace it is all the more surprising that very little can be learnt about its exact nature except perhaps that it is some kind of similarity or analogy. There is, however, a lot more to it than that. Indeed, as we shall try to show below, the general ideas underlying the three central theorems concerning internal limitations of formal deductive systems can be taken as different ways to resolve the Liar paradox. More precisely, it will turn out that an abstract formal variant of the Liar paradox, which can almost straightforwardly be inferred from its original ordinary language version, is a possible common generalization of (both the syntactic and semantic versions of) Gödel's incompleteness theorem, the theorem of Tarski on the undefinability of truth, and that of Church concerning the undecidability of provability. We feel that, in addition to its outcome, the process of deriving abstract forms of such important mathematical results from a common language expression is in itself interesting and enlightening. On the other hand, our investigations will shed some light on the limits one cannot get beyond solely on the basis of the Liar paradox.

To obtain the formal resolution of the Liar paradox that can be considered as the common generalization of the theorems concerned, we shall reformulate it in a step-by-step manner in four main stages. First we shall seek an ordinary language equivalent of the paradox in a form that shows clearly its logical structure, and then we shall directly translate the expression we have

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