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SCATTERING FOR NONLINEAR SYMMETRIC HYPERBOLIC SYSTEMS

By

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0 Introduction

In this paper we shall investigate the Cauchy problem and scattering for the following nonlinear symmetric hyperbolic system of first order

$$E(u)\frac{\partial u}{\partial t}=\sum_{j=1}^{n}A_j(u)\frac{\partial u}{\partial x_j}+F(t,x,u), \tag{0.1}$$

where $x\in\boldsymbol{R}^n$, $t\in\boldsymbol{R}^1$, $u=u(t,x)$ is a real $m\times 1$ matrix. $E(u)$ is an $m\times m$ matrix which is real, symmetric and positive definite, $A_j(u)$ $(j=1,\dots,n)$ are $m\times m$ matrices which are real and symmetric. Moreover we assume that $E(u),A_j(u)$, $F(u)\in C^\infty(\boldsymbol{R}^m)$.

First, in order to obtain the existence of the time global solution of the Cauchy problem for the equation (0.1), we consider the following Cauchy problem for a linear symmetric hyperbolic system of first order with constant coefficients;

$$\begin{cases} E^0\dfrac{\partial u^0}{\partial t}=\displaystyle\sum_{j=1}^{n}A_j^0\frac{\partial u^0}{\partial x_j}, \\ u^0(0,x)=\varphi_0(x), \end{cases} \tag{0.2}$$

where $x\in\boldsymbol{R}^n$, $t\in\boldsymbol{R}^1$, $u^0=u^0(t,x)$ and $\varphi_0(x)$ are real $m\times 1$ matrices and $\varphi_0(x)\in C_0^\infty(\boldsymbol{R}^n)$. E^0 is a $m\times m$ matrix which is real, symmetric and positive definite. A_j^0 $(j=1,\dots,n)$ are $m\times m$ matrices which are real, symmetric and constant. We assume that the eigenvalues $\lambda_j(\xi)$ of $\sum_{j=1}^{n}A_j^0\xi_j$ are non zero, real, distinct and their slowness surfaces are strictly convex. S. Lucente and G. Ziliotti [1] obtain the decay estimate of the solutions of the Cauchy problem (0.2) as $t\to\pm\infty$. By using their estimate and the existence of the local solution (cf: [3]),

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