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INTEGRAL GEOMETRY ON PRODUCT OF SPHERES

By

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1. Introduction and Result

One of the oldest results in integral geometry is the Poincaré formula for the average of the intersection number of two curves. Many differential geometers have studied the Poincaré formula from various points of view. In particular, R. Howard [1] generalized this formula in Riemannian homogeneous spaces and obtained the following formula.

THEOREM 1.1 [1]. *Let G/K be a Riemannian homogeneous space with a G-invariant Riemannian metric, and let M and N be submanifolds of G/K with* $\dim M + \dim N = \dim(G/K)$. *Assume that G is unimodular and for almost all $g \in G$, M and gN intersect transversely. Then*

$$\int_G \sharp(M \cap gN)\, d\mu_G(g) = \int_{M \times N} \sigma_K(T_x^{\perp}M, T_y^{\perp}N)\, d\mu_{M \times N}(x, y),$$

where $\sharp(X)$ denotes the number of points in X and $\sigma_K(T_x^{\perp}M, T_y^{\perp}N)$ is defined by (2.1) *below.*

This theorem plays an important role in this paper. In the case that $G/K = \boldsymbol{R}^2$, this formula implies the classical Poincaré's one. In the case that G/K is a space of constant curvature, the isotropy group K acts transitively on the Grassmann manifolds consisting of subspaces in $T_o(G/K)$, so $\sigma_K(T_x^{\perp}M, T_y^{\perp}N)$ on the right side of the above integral in Theorem 1.1 is constant. Namely, $\sigma_K(V, W)$ is independent on V and W. Hence we can have clearly expressed $\sigma_K(T_x^{\perp}M, T_y^{\perp}N)$, that is,

$$\sigma_{SO(n)}(T_x^{\perp}M^p, T_y^{\perp}N^q) = \frac{\mathrm{vol}(S^0)\,\mathrm{vol}(SO(n+1))}{\mathrm{vol}(S^p)\,\mathrm{vol}(S^q)}.$$

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