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KENMOTSU TYPE REPRESENTATION FORMULA FOR SPACELIKE SURFACES IN THE DE SITTER 3-SPACE

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Introduction

In [10], Kenmotsu proved that surfaces in the Euclidean 3-space $\boldsymbol{E}^3$ can be represented by means of the mean curvature and the Gauss map. In [3] and [4], we gave the Kenmotsu type representation formulas for surfaces in the hyperbolic 3-space (cf. [11]) and the Riemannian 3-sphere. For each Riemannian 3-space form N^3 and a surface M^2 in N^3, we can consider an adapted frame on M^2 as a map from M^2 to the isometry group $\mathrm{Isom}(N^3)$. The 'Gauss map' of M^2 to $\boldsymbol{S}^2(=SO(3)/SO(2))$ is defined from the 'rotational part' (i.e., $SO(3)$-part) of the adapted framing map. (For example, $\mathrm{Isom}(\boldsymbol{E}^3)=\boldsymbol{R}^3\rtimes SO(3)$.)

On the other hand, Nishikawa and the second author [8] proved the Lorentzian version of the Kenmotsu representation formula for spacelike surfaces in the Minkowski 3-space $\boldsymbol{L}^3$ (cf. [12]). Here $\mathrm{Isom}(\boldsymbol{L}^3)=\boldsymbol{R}^3\rtimes SO_0(1,2)$ and hence the Gauss map is a map to the upper hyperboloid $\boldsymbol{H}^2$ $(=SO_0(1,2)/SO(2))$. In this paper, we introduce the Kenmotsu type representation formula for spacelike surfaces in the Lorentzian 3-space form of constant curvature 1, that is, the de Sitter 3-space $\boldsymbol{S}_1^3$. A similar formula in the anti-de Sitter 3-space has been already given in [6].

1. De Sitter 3-space $\boldsymbol{S}_1^3$

The de Sitter 3-space $\boldsymbol{S}_1^3$ is defined as the semi-sphere in the Minkowski 4-space $\boldsymbol{L}^4$ of radius 1. As in [9] and [1], it is convenient to use the complex special linear group $SL(2;\boldsymbol{C})$, which is the double cover of $SO_0(1,3)$, as the group of isometries of $\boldsymbol{S}_1^3$. Put

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