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ON FOURIER COEFFICIENTS OF MAASS WAVE FORMS OF HALF INTEGRAL WEIGHT BELONGING TO KOHNEN'S SPACES

By

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Introduction

Waldspurger [15] proved that the squares of Fourier coefficients $a(n)$ at a square free integer n of a modular form $f(z)=\sum_{n=1}^{\infty}a(n)e[nz]$ of half integral weight are essentially proportional to the critical value of the zeta function at a certain integer attached to the modular form F of even integral weight if f corresponds to F by the Shimura correspondence Ψ and f is an eigen-function of Hecke operators. Kohnen-Zagier [2], [4] determined explicitly the constant of the proportionality in the case of modular forms of belonging to Kohnen's spaces $S_{(2k+1)/2}(N,\chi)$ of weight $(2k+1)/2$ and of square free level N with character χ which is a subspace of $S_{(2k+1)/2}(4N,\chi_1)$, where $S_{(2k+1)/2}(4N,\chi_1)$ means the space of modular cusp forms of half integral weight given in [10]. Kohnen-Zagier [2] (resp. Kohnen [4]) treated the case where $N=1$ (resp. N is an odd square free integer and χ is the trivial character of level N) (cf. Kojima [5] and [7]).

In [10], Shimura intended to generalize such formulas to the case of Hilbert modular forms f of half integral weight and succeeded in obtaining many general interesting formulas. Among these, some explicit and useful formulas about the proportionality constant were formulated under assumptions that f satisfies the multiplicity one theorem. K. K-Makdisi [1] gave a generalization of these to the case of Hilbert-Maass wave forms. For modular forms belonging to Kohnen's spaces, they did not obtain the same explicit formulas as those of Kohnen and Zagier [2], [4].

In [8], we derived such explicit formulas concerning the proportionality constant in some cases of modular forms f of half integral weight whose multiplicity are two and generalized results of Kohnen and Zagier in [2], [4] to

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