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ON THE FIRST EIGENVALUE OF NON-ORIENTABLE CLOSED SURFACES

By

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0. Introduction

Let (M, g) be a 2-dimensional non-orientable closed Riemannian manifold. We study the spectrum of the Laplacian for functions on (M, g). We express it by

$$\mathrm{Spec}(M, g) = \{0 = \lambda_0 < \lambda_1 \leq \lambda_2 \leq \lambda_3 \leq \cdots\}.$$

Let $(\tilde{M}, \tilde{g})$ be the orientable Riemannian double cover of (M, g). Our interest is what properties are preserved between (M, g) and $(\tilde{M}, \tilde{g})$. The positive first eigenvalue $\lambda_1(M, g)$ has many geometric informations. We have interests in the influences for the positive first eigenvalue by taking the Riemannian double cover. Generally we have $\lambda_1(M, g) \geq \lambda_1(\tilde{M}, \tilde{g})$. So we study the difference between $\lambda_1(M, g)$ and $\lambda_1(\tilde{M}, \tilde{g})$. Especially we find the cases that $\lambda_1(M, g) = \lambda_1(\tilde{M}, \tilde{g})$ holds good.

It is well-known (cf. [9]) that 2-dimensional closed manifolds are classified as follows.

The Classification Theorem of Closed Surfaces. *A closed surface is homeomorphic to one of the following spaces.*

$$S^2, T^2, \#^n T^2 \ (n \geq 2) : \textit{orientable}$$

$$\boldsymbol{RP}^2, \#^n \boldsymbol{RP}^2 \ (n \geq 2) : \textit{non-orientable}$$

where $\#^n M$ means the connected sum of n-copies of a manifold M. Moreover the double cover of $\#^n \boldsymbol{RP}^2$ $(n \geq 2)$ is homeomorphic to $\#^{n-1} T^2$.

In this paper we show the following results.

THEOREM A. *If M is homeomorphic to $\boldsymbol{RP}^2$, then*

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