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ON A CLASS OF SELF-INJECTIVE LOCALLY BOUNDED CATEGORIES

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Throughout the paper K denotes a fixed algebraically closed field. Let R be a locally bounded K-category in the sense of [3]. It is well-known that every locally bounded K-category R is isomorphic to a factor category KQ_R/I_R, where KQ_R is a path category of a locally-finite quiver and I_R is some admissible ideal in KQ_R. A locally bounded K-category $R \cong KQ_R/I_R$ is said to be *triangular* if Q_R has no oriented cycles.

For a locally bounded K-category R we denote by $\mathrm{mod}(R)$ the category of all finite-dimensional right R-modules.

We are interested in self-injective locally bounded K-categories. Assume that R is a self-injective locally bounded triangular K-category which is connected. Then there is the Nakayama K-automorphism $\nu_R : R \to R$ which is induced by a permutation π_R of the isoclasses of simple right R-modules such that $\pi_R(\mathrm{top}(P)) = \mathrm{soc}(P)$ for every indecomposable projective right R-module P. Consequently, the infinite cyclic group (ν_R) generated by the Nakayama automorphism ν_R acts freely on the objects of R. We consider self-injective, locally bounded, triangular and connected K-categories R whose quotient categories $R/(\nu_R)$ are finite-dimensional K-algebras and there is no indecomposable projective R-module of length smaller than 3.

Every basic finite-dimensional K-algebra A can be considered as a locally bounded K-category, because $A \cong KQ_A/I_A$ for a finite quiver Q_A. The *repetitive category* (see [5]) of a basic finite-dimensional K-algebra A is the self-injective locally bounded K-category $\hat{A}$ whose objects are formed by the pairs $(z, x) = x_z$, $x \in ob(A)$, $z \in \boldsymbol{Z}$ and $\hat{A}(x_z, y_z) = \{z\} \times A(x, y)$, $\hat{A}(x_{z+1}, y_z) = \{z\} \times DA(y, x)$, and $\hat{A}(x_p, y_q) = 0$ if $p \neq q$, $q+1$, where DV denotes the dual space $\mathrm{Hom}_K(V, K)$. It is well-known that if A is triangular then $\hat{A}$ is triangular. Moreover, $\hat{A}/(\nu_{\hat{A}})$ is

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