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A FAMILY OF BRAIDED COSEMISIMPLE HOPF ALGEBRAS OF FINITE DIMENSION

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0. Introduction

Recently some finite dimensional cosemisimple Hopf algebras were constructed [Mas2] [F] [G]. We aim to give a plain and systematic description of cosemisimple Hopf algebras of low dimension. For this purpose we construct them as quotient bialgebras of a sufficiently large bialgebra. This way has the advantage of defining homomorphisms and determining braidings.

In this paper we define and study a family of finite dimensional cosemisimple Hopf algebras

$$\mathscr{F} = \{A_{NL}^{(++)}, A_{NL}^{(+-)}, A_{NL}^{(-+)}, A_{NL}^{(--)} \mid N \geqq 1, L \geqq 2\},$$

which consists of quotients of a bialgebra B over an algebraically closed field k with $chk \neq 2$.

This family contains the "non-trivial" cosemisimple Hopf algebras of dimension 8, 12 if $chk \neq 3$.

In Section 1 we review basic definitions and results.

In Section 2 quadratic bialgebras B, $B^{(+)}$ and $B^{(-)}$ are constructed. We use B to construct the family $\mathscr{F}$, and $B^{(\pm)}$ to obtain braidings on the members of a subfamily of $\mathscr{F}$. These bialgebras B, $B^{(\pm)}$ are cosemisimple, and we determine all braidings on them.

In Section 3 we define the family $\mathscr{F}$ as a set of quotient bialgebras of the bialgebra B. We write $A_{NL}^{(+1,-1)} = A_{NL}^{(+-)}$, etc. Let $\nu, \lambda = \pm 1$. Our main results are as follows.

i) $A_{NL}^{(\nu\lambda)}$ is a non-cocommutative involutory cosemisimple Hopf algebra of dimension $4NL$, which is non-commutative unless $(L, \lambda) = (2, +1)$. $A_{NL}^{(\nu\lambda)}$ is furthermore semisimple if $(dim\, A_{NL}^{(\nu\lambda)}) \cdot 1 \neq 0$.

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