

INDUCED MAPPINGS ON HYPERSPACES II

By

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Abstract. A mapping $f: X \rightarrow Y$ between continua induces a mapping $C(f): C(X) \rightarrow C(Y)$ between hyperspaces. In section 2, we shall give a condition under which $C(f)$ becomes confluent whenever f is confluent. In section 3, we consider the image and the inverse image of an order arc under the mapping $C(f)$ and characterizations of a confluent mapping and a hereditarily confluent mapping. In the last section, we treat about particular subcontinua (which are like fat Whitney levels) and inverse image of them under $C(f)$.

1. Introduction

In this paper, continua are compact connected metric spaces and mappings are continuous functions. The letters X and Y will always denote nondegenerate continua. The *hyperspaces* of X are the metric spaces $2^X = \{K \subset X: K \text{ is nonempty and compact}\}$ and $C(X) = \{K \in 2^X: K \text{ is connected}\}$ with the Hausdorff metric H_d (see [10] for the definition of the Hausdorff metric and basic properties of hyperspaces). A mapping $f: X \rightarrow Y$ induces a mapping $C(f): C(X) \rightarrow C(Y)$, where $C(f)(K) = f(K)$ for each $K \in C(X)$. Clearly, $C(g \circ f) = C(g) \circ C(f)$. In [4], we have proved that if Y is locally connected and f is confluent, then $C(f)$ is confluent. In section 2, we show that the condition of locally connectedness can be weakened (Theorem 2.7).

An *order arc* in $C(X)$ is an arc $\alpha \subset C(X)$ such that for $K, L \in \alpha$, $K \subset L$ or $L \subset K$. An order arc with the end points X and $\{x\}$ for some $x \in X$ is called a *large order arc*. Let $\Gamma(X) = \{\alpha: \alpha \text{ is an order arc in } C(X)\} \cup F_1(X)$, where

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