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ALMOST KÄHLER STRUCTURES ON THE RIEMANNIAN PRODUCT OF A 3-DIMENSIONAL HYPERBOLIC SPACE AND A REAL LINE

By

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1. Introduction.

An almost Hermitian manifold $M=(M,J,g)$ is called an almost Kähler manifold if the Kähler form is closed (or equivalently $\mathfrak{S}_{x,y,z} g\ \nabla_x J\ Y,Z\ = 0$ for $X,Y,Z \in \mathfrak{X}(M)$, where $\mathfrak{S}$ and $\mathfrak{X}(M)$ denotes the cyclic sum and the Lie algebra of all differentiable vector fields on M respectively). A Kähler manifold, which is defined by $\nabla J = 0$, is necessarily an almost Kähler manifold. A non-Kähler almost Kähler manifold is called a strictly almost Kähler manifold. It is well-known that an almost Kähler manifold with integrable almost complex structure is a Kähler manifold. Concerning the integrability of almost Kähler manifolds, the following conjecture by S. I. Goldberg is known ([1]):

CONJECTURE. *A compact almost Kähler Einstein manifold is a Kähler manifold.*

The second author has proved that the above conjecture is true for the case where the scalar curvature is nonnegative ([4]). However, the above conjecture is still open in the case where the scalar curvature is negative. Recently, the authors proved that a $2n(\geq 4)$-dimensional hyperbolic space $\boldsymbol{H}^{2n}$ cannot admits (compatible) almost Kähler structure ([3]).

In the present paper, we consider about (compatible) almost Kähler structures on the Riemannian product $\boldsymbol{H}^3 \times \boldsymbol{R}$ of a 3-dimensional hyperbolic space $\boldsymbol{H}^3$ and a real line $\boldsymbol{R}$. We construct an example of strictly almost Kähler structure (J,g) on the Riemannian product $\boldsymbol{H}^3 \times \boldsymbol{R}$ and determine the automorphism group of the almost Kähler manifold $(\boldsymbol{H}^3 \times \boldsymbol{R}, J, g)$. To our knowledge, this is the first example of strictly almost Kähler symmetric space. Moreover, we prove that the Riemannian product $\boldsymbol{H}^3 \times \boldsymbol{R}$ provided with a (compatible) almost Kähler structure (J,g) cannot be a universal (almost Hermitian) covering of any compact almost Kähler manifold (Theorem 2 in

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