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ON THE SOLVABILITY OF CONVOLUTION EQUATIONS IN $\mathcal{K}'_M$

By

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Abstract. Let $\mathcal{K}'_M$ be the space of distributions on R^n which grow no faster than $e^{M(kx)}$ for some $k>0$ where M is an increasing continuous function on R^n, and let $\mathcal{O}'_C(\mathcal{K}'_M; \mathcal{K}'_M)$ be the space of convolution operators in $\mathcal{K}'_M$. We show that, for $S\in\mathcal{O}'_C(\mathcal{K}'_M; \mathcal{K}'_M)$, $S*\mathcal{K}'_M=\mathcal{K}'_M$ is equivalent to the following: Every distribution $u\in\mathcal{O}'_C(\mathcal{K}'_M; \mathcal{K}'_M)$ with $S*u\in\mathcal{K}_M$ is in $\mathcal{K}_M$.

1. Introduction.

Let $\mathcal{K}'_M$ be the space of distributions on R^n which grow no faster than $e^{M(kx)}$ for some $k>0$, where M is an increasing continuous functions on R^n; $\mathcal{K}'_M$ is the dual space of $\mathcal{K}_M$, which we describe later. We denote by $\mathcal{O}'_C(\mathcal{K}'_M; \mathcal{K}'_M)$ the space of convolution operators in $\mathcal{K}'_M$.

In [1], S. Abdullah proved that, if S is a distributions in $\mathcal{O}'_C(\mathcal{K}'_M; \mathcal{K}'_M)$ and $\hat{S}$ is its Fourier transform, the following conditions are equivalent:

(a) There exist positive constants A, C and a positive integer N such that

$$\sup_{\substack{z\in C^n \\ |z|\leq A\Omega^{-1}(\log(2+|\xi|))}} |\hat{S}(z+\xi)| \geq \frac{C}{(1+|\xi|)^N}, \qquad \xi\in R^n$$

where Ω^{-1} is the inverse of Ω, which is the dual to M in the sense of Young.

(b) $S*\mathcal{K}'_M=\mathcal{K}'_M$.

In this paper we prove that, for $S\in\mathcal{O}'_C(\mathcal{K}'_M; \mathcal{K}'_M)$, the statements (a) and (b) are equivalent to the following: Every distribution $u\in\mathcal{O}'_C(\mathcal{K}'_M; \mathcal{K}'_M)$ satisfying $S*u\in\mathcal{K}_M$ is in $\mathcal{K}_M$.

The motivation for this problem comes from the paper [5]. Here S. Sznaider and Z. Zielezny proved that, if S is a distribution in $\mathcal{O}'_C(\mathcal{K}'_1; \mathcal{K}'_1)$ and $\hat{S}$ is its Fourier transform, the following statements are equivalent:

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