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ON A FAMILY OF QUOTIENTS OF FERMAT CURVES

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Introduction

Let F_N be the N-th Fermat curve defined by the equation:

$$u^N+v^N=1.$$

For a pair (r, s) of positive integers such that $r+s\leqq N-1$ and g.c.d. $(r, s, N)=1$, we denote by $F(r, s)$ the quotient of F_N defined by the equation:

$$y^N=x^r(1-x)^s$$

where the projection $F_N\to F(r, s)$ is defined by

$$(x, y)\longmapsto(u^N, u^r v^s).$$

We denote by $\sigma(r, s)$ the automorphism of $F(r, s)$ defined by $\sigma(r, s)^*: (x, y)\mapsto(x, \zeta_N y)$ where ζ_N is a primitive N-th root of unity. The order N of $\sigma(r, s)$ is quite large for the genus $g(r, s)$ of $F(r, s)$. Between them we have a relation:

$$(\sharp)\qquad N\geqq 2g(r, s)+1.$$

Conversely the inequality ($\sharp$) characterize the quotients $F(r, s)$. In fact we have the following (cf. Theorem 2.2):

THEOREM. *Let X be a complete non-singular curve of genus g over an algebraically closed field k of characteristic* 0, *and let σ be an automorphism of X of order N with $N\geqq 2g+1\geqq 5$. Let H_λ be a hyperelliptic curve of genus g defined by the equation $y^2=(x^{g+1}-1)(x^{g+1}-\lambda)$ with $\lambda\in k\setminus\{0, 1\}$, and let τ_λ be an automorphism of H_λ defined by $\tau_\lambda^*: (x, y)\mapsto(\zeta_{g+1}x, -y)$. Assume that the pair (X, σ) is not isomorphic to $(H_\lambda, \langle\tau_\lambda\rangle)$ for any λ with $N=2g+2$ and g even. Then the pair (X, σ) is isomorphic to $(F(r, s), \sigma(r, s))$, for some (r, s).*

In this paper we are mainly concerned with the curves $F(r, s)$ in which the equality $N=2g(r, s)+1$ holds in ($\sharp$). In a family of these curves there are some interesting curves. For example we have a curve whose group of automor-

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