

ON REAL HYPERSURFACES OF TYPE A IN A COMPLEX SPACE FORM (I)

By

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§ 1. Introduction.

A complex n -dimensional Kähler manifold of constant holomorphic sectional curvature c is called a *complex space form*, which is denoted by $M_n(c)$. A complete and simply connected complex space form consists of a complex projective space $P_n\mathbb{C}$, a complex Euclidean space \mathbb{C}^n or a complex hyperbolic space $H_n\mathbb{C}$, according as $c > 0$, $c = 0$ or $c < 0$.

Now, let M be a real hypersurface of an n -dimensional complex space form $M_n(c)$. Then M has an almost contact metric structure (ϕ, ξ, η, g) induced from the Kähler metric and the almost complex structure of $M_n(c)$. Okumura [7] and Montiel and Romero [6] proved the following

THEOREM A. *Let M be a real hypersurface of $P_n\mathbb{C}$, $n \geq 2$. If it satisfies*

$$(1.1) \quad A\phi - \phi A = 0,$$

then M is locally a tube of radius r over one of the following Kähler submanifolds:

(A₁) *a hyperplane $P_{n-1}\mathbb{C}$, where $0 < r < \pi/2$,*

(A₂) *a totally geodesic $P_k\mathbb{C}$ ($1 \leq k \leq n-2$), where $0 < r < \pi/2$,*

where A is the shape operator in the direction of the unit normal C on M .

THEOREM B. *Let M be a real hypersurface of $H_n\mathbb{C}$, $n \geq 2$. If it satisfies (1.1), then M is locally one of the following hypersurfaces:*

(A₀) *a horosphere in $H_n\mathbb{C}$, i. e., a Montiel tube,*

(A₁) *a tube of a totally geodesic hyperplane $H_{n-1}\mathbb{C}$,*

(A₂) *a tube of a totally geodesic $H_k\mathbb{C}$ ($1 \leq k \leq n-2$).*

On the other hand, the following theorem is proved by Maeda and Udagawa [4] under that the structure vector ξ is principal and then recently by Kimura

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