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A RELATION BETWEEN k-th UV^{k+1} GROUPS AND k-th STRONG SHAPE GROUPS

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1. Introduction

Compacta X and Y are *UV^n-equivalent* provided that there exist sequences $\{E_i\}_{1\leq i\leq m}$ and $\{F_i\}_{0\leq i\leq m}$ of compacta and sequences $\{f_i\}_{1\leq i\leq m}$ and $\{g_i\}_{1\leq i\leq m}$ of UV^n-maps $f_i: E_i\to F_{i-1}$ and $g_i: E_i\to F_i$, where $F_0=X$ and $F_m=Y$. Replacing UV^n-maps with CE-maps, we have the definition of CE-equivalence.

$$\begin{tikzcd}
& E_1 \arrow[dl, "f_1"'] \arrow[dr, "g_1"] & & \cdots \arrow[dl] \arrow[dr] & & E_m \arrow[dl, "f_m"'] \arrow[dr, "g_m"] & \\
X=F_0 & & F_1 & & F_{m-1} & & F_m=Y
\end{tikzcd}$$

It is well known that finite-dimensional CE-equivalent compacta are shape equivalent (see [D-S]). The first example that shows the gap between shape equivalence and CE-equivalence was found by Ferry [Fe1]. In [Fe3], it was shown that UV^m-equivalent n-dimensional compacta are shape equivalent. Next Daverman and Venema [D-V] constructed an n-dimensional LC^{n-2}-continuum which is shape equivalent but not UV^{n-1}-equivalent to S^1. Mrozik [Mr1] obtained a method to have continua which are shape equivalent but not UV^1-equivalent to each other. Moreover Mrozik [Mr2] improved the method and had a strategy to construct a LC^n-continuum Y from any LC^{n+1}-continuum X with $\pi_1(X)$ infinite such that they are shape equivalent but not UV^{n+1}-equivalent. As a criterion of UV^n-equivalence he introduced the notions of UV^n-component $\pi_0^{(n)}(X)$ [Mr1], k-th UV^n-homotopy group $\pi_k^{(n)}(X)$ and k-th CE-homotopy group $\pi_k^{CE}(X)$ [Mr2]. Venema [Ve] investigated the groups and showed that $\pi_k^{(k+1)}(X)=\pi_k^{(k+2)}(X)=\cdots=\pi_k^{CE}(X)$ for every continuum X and that $\pi_n^{(n)}(Y)=0$ for every UV^n-continuum Y.

In this paper we consider a relation between $\pi_k^{(k+1)}(X)$ and the k-th strong shape group $\underline{\underline{\pi}}_k(X)$ [Q]. We define a natural homomorphism $s_k: \pi_k^{(k+1)}(X)\to\underline{\underline{\pi}}_k(X)$ and show that, if pro-$\pi_1(X)$ is profinite, s_k is an isomorphism. As its

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