

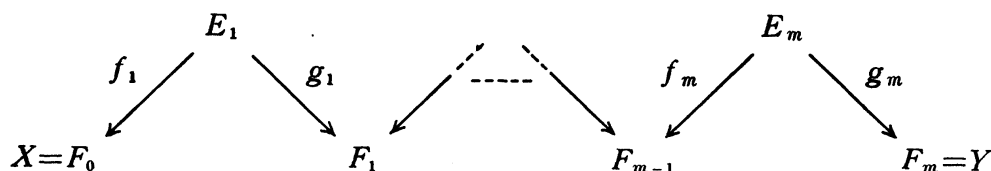
A RELATION BETWEEN k -th UV^{k+1} GROUPS AND k -th STRONG SHAPE GROUPS

By

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1. Introduction

Compacta X and Y are UV^n -equivalent provided that there exist sequences $\{E_i\}_{1 \leq i \leq m}$ and $\{F_i\}_{0 \leq i \leq m}$ of compacta and sequences $\{f_i\}_{1 \leq i \leq m}$ and $\{g_i\}_{1 \leq i \leq m}$ of UV^n -maps $f_i: E_i \rightarrow F_{i-1}$ and $g_i: E_i \rightarrow F_i$, where $F_0 = X$ and $F_m = Y$. Replacing UV^n -maps with CE -maps, we have the definition of CE -equivalence.



It is well known that finite-dimensional CE -equivalent compacta are shape equivalent (see [D-S]). The first example that shows the gap between shape equivalence and CE -equivalence was found by Ferry [Fe1]. In [Fe3], it was shown that UV^m -equivalent n -dimensional compacta are shape equivalent. Next Daverman and Venema [D-V] constructed an n -dimensional LC^{n-2} -continuum which is shape equivalent but not UV^{n-1} -equivalent to S^1 . Mrozik [Mr1] obtained a method to have continua which are shape equivalent but not UV^1 -equivalent to each other. Moreover Mrozik [Mr2] improved the method and had a strategy to construct a LC^n -continuum Y from any LC^{n+1} -continuum X with $\pi_1(X)$ infinite such that they are shape equivalent but not UV^{n+1} -equivalent. As a criterion of UV^n -equivalence he introduced the notions of UV^n -component $\pi_0^{(n)}(X)$ [Mr1], k -th UV^n -homotopy group $\pi_k^{(n)}(X)$ and k -th CE -homotopy group $\pi_k^{CE}(X)$ [Mr2]. Venema [Ve] investigated the groups and showed that $\pi_k^{(k+1)}(X) = \pi_k^{(k+2)}(X) = \dots = \pi_k^{CE}(X)$ for every continuum X and that $\pi_n^{(n)}(Y) = 0$ for every UV^n -continuum Y .

In this paper we consider a relation between $\pi_k^{(k+1)}(X)$ and the k -th strong shape group $\underline{\pi}_k(X)$ [Q]. We define a natural homomorphism $s_k: \pi_k^{(k+1)}(X) \rightarrow \underline{\pi}_k(X)$ and show that, if $\text{pro-}\pi_1(X)$ is profinite, s_k is an isomorphism. As its

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