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UNIQUENESS AND EXISTENCE OF DUALITIES OVER COMPACT RINGS

Dedicated to Santuzza Ghezzo Baldassarri

By

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0. Introduction

0.1. In relatively recent times it has been proved that some classical dualities between additive categories are unique. For example, Roeder [13] proved that any duality on the category of locally compact abelian groups coincides, up to natural equivalences, with the Pontryagin duality.

Inspired by this fact, I. Prodanov [11] held at Sofia University, at the end of the 70's, a seminar on dualities and spectral spaces, suggesting some similar results: we recall those by Dimov [2], Stoyanov [14] and the first author [4].

Dimov proved that Stone duality is the unique duality between the category of Hausdorff compact totally disconnected spaces and the category of Boolean rings.

Let (A, σ) be a compact ring and denote by $\mathcal{L}\text{-}A_\sigma$ $(A_\sigma\text{-}\mathcal{L})$ the category of locally compact right (left) topological modules over (A, σ). L. Stoyanov proved that, if A is commutative, then the unique duality between $\mathcal{L}\text{-}A_\sigma$ and $A_\sigma\text{-}\mathcal{L}$ is the Pontryagin duality, by using the Theorem of Kaplansky and Zelinsky on the decomposition of a *commutative* compact ring as a product of local rings.

Stoyanov's theorem has been extended by the first author [4] to the non commutative case, by using his results on equivalences between closed categories of modules [5].

Unfortunately the activity of Ivan Prodanov, who inspired this line of research, was interrupted by his untimely death in April 1985.

0.2. If we use the result of Stoyanov and Gregorio, it is easy to show that if (A, σ) and (R, τ) are compact rings, then, if a duality $H=(H_1, H_2)$

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