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COVERING PROPERTIES IN COUNTABLE PRODUCTS

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1. Introduction.

A space X is said to be *subparacompact* if every open cover of X has a σ-discrete closed refinement, and *metacompact* (*countably metacompact*) if every open cover (countable open cover) of X has a point finite open refinement. A space X is said to be *metalindelöf* if every open cover of X has a point countable open refinement. A collection $\mathcal{U}$ of subsets of a space X is said to be *interior-preserving* if $\mathrm{int}(\cap\mathcal{V})=\cap\{\mathrm{int}\,V : V\in\mathcal{V}\}$ for every $\mathcal{V}\subset\mathcal{U}$. Clearly, an open collection $\mathcal{U}$ is interior-preserving if and only if $\cap\mathcal{V}$ is open for every $\mathcal{V}\subset\mathcal{U}$. A space X is said to be *orthocompact* if every open cover of X has an interior-preserving open refinement. Every paracompact Hausdorff space is subparacompact and metacompact, and every metacompact space is countably metacompact, metalindelöf and orthocompact. The reader is refered to D. K. Burke [4] for a complete treatment of these covering properties and some informations of their role in general topology.

Let $\mathcal{DC}$ be the class of all spaces which have a discrete cover by compact sets. The topological game $G(\mathcal{DC}, X)$ was introduced and studied by R. Telgársky [19]. The games are played by two persons called Players I and II. Players I and II choose closed subsets of II's previous play (or of X, if $n=0$): Player I's choice must be in the class $\mathcal{DC}$ and II's choice must be disjoint from I's. We say that Player I *wins* if the intersection of II's choices is empty. Recall from [19] that a space X is said to be *$\mathcal{DC}$-like* if Player I has a winning strategy in $G(\mathcal{DC}, X)$. The class of $\mathcal{DC}$-like spaces includes all spaces which admit a σ-closure-preserving closed cover by compact sets, and regular subparacompact, σ-C-scattered spaces.

Paracompactness and Lindelöf property of countable products have been studied by several authors. In particular, if X is a separable metric space or X is a regular Čech-complete Lindelöf space or X is a regular C-scattered Lindelöf space, then $X^{\omega}\times Y$ is Lindelöf for every regular hereditarily Lindelöf space Y. The first result is due to E. Michael (cf. [14]) and the second one

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