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THE CYCLIC EXTENSIBILITY OF ESSENTIAL COMPONENTS OF THE FIXED POINT SET

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1. Introduction.

All spaces considered in this paper are separable metric and every mapping is continuous unless otherwise stated. Let X be a continuum[1]. If every continuous mapping $f: X\rightarrow X$ has at least one fixed point, X is called to have the *fixed point property* (*f.p.p.*). In this paper we investigate the existence of essential components of the fixed point sets and the property f.*p.p., which are defined as follows: a component C of the fixed point set of f is called *essential*, if for any $\varepsilon>0$ there exists $\delta>0$ such that every continuous mapping $f': X\rightarrow X$ with $|f'-f|<\delta$ has a fixed point in the ε-neighborhood $U_{\varepsilon}(C)$ of C, and if otherwise it is called *non-essential*; and X has *f.*p.p.*, if X has f.p.p., and the fixed point set of every continuous mapping $f: X\rightarrow X$ has at least one essential component (see [2], [7]). Note that there exists a space which has f.p.p., but does not have f.*p.p. (see [6]).

The Hilbert cube I^{ω} has f.*p.p. and the property f.*p.p. is invariant under retractions. Hence every compact absolute retract has f.*p.p. (see [2]). Further, if X and Y are two continua with f.*p.p. and $X\cap Y$ is a single point, then $X\cup Y$ has f.*p.p. (see [1], [4], [5]). The last statement has been extended to the special case where the number of continua is countably infinite (see [5]). The purpose of this paper is to extend the above property to a more general setting; we prove that a continuum X has f.*p.p. whenever it can be expressed as the union of a null sequence of subcontinua X_{α}'s with f.*p.p. such that any pair of X_{α} and X_{β} $(\alpha\neq\beta)$ has at most one point in common and that the boundary of each component of $X-X_{\alpha}$ consists of a single point for every α (see the Main Theorem). When X is locally connected, it means the cyclic extensibility of f.*p.p. (see [3], [4] and the Corollary).

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1) A continuum means a compact, connected metric space.