TSUKUBA J. MATH.
Vol. 17 No. 2 (1993), 497–512

COMPLETE SPACE-LIKE SUBMANIFOLDS WITH PARALLEL MEAN CURVATURE VECTOR OF AN INDEFINITE SPACE FORM

By

Qing-ming CHENG* and Soon Meen CHOI

1. Introduction.

Let $M_p^{n+p}(c)$ be an $(n+p)$-dimensional connected indefinite Riemannian manifold of index p and of constant curvature c, which is called *an indefinite space form of index p*. According to $c>0$, $c=0$ or $c<0$ it is denoted by $S_p^{n+p}(c)$, $\boldsymbol{R}_p^{n+p}$ or $H_p^{n+p}(c)$. A submanifold M of an indefinite space form $M_p^{n+p}(c)$ is said to be *space-like* if the induced metric on M from that of the ambient space is positive definite. It is pointed out by some physicians that space-like hypersurfaces with constant mean curvature of arbitrary spacetimes get interested in relativity theory and an entire space-like hypersurface with constant mean curvature of an indefinite space form are studied by many authors (for examples: [1], [2], [3], [4], [7], [12] and so on).

Now, for a complete space-like submanifold M with parallel mean curvature vector of $S_p^{n+p}(c)$, it is also seen by the first author [5] that M is totally umbilic if $n=2$ and $h^2\leqq 4c$ or if $n>2$ and $h^2<4(n-1)c$, where H denotes the mean curvature, i.e., the norm of the mean curvature vector and $h=nH$. On the other hand, the first author and Nakagawa [6] investigated the total umbilicness of such hypersurfaces from the different point of view. They proved that the squared norm S of the second fundamental form of M is bounded from above by $S_+(1)$ and if sup $S<S_-(1)$ and $H^2\leqq c$, then M is totally umbilic, where

$$S_\pm(p)=-pnc+\frac{nh^2\pm(n-2)\sqrt{h^4-4(n-1)ch^2}}{2(n-1)}.$$

In this paper, we research the similar problem to the above property for the complete space-like submanifolds with parallel mean curvature vector of an indefinite space form. That is, we prove the following

* The project is supported by National Natural Science Foundation of China.
Received November 5, 1992.