TSUKUBA J. MATH.
Vol. 17 No. 1 (1993), 193—200

SOME BOUNDS FOR THE SPECTRAL RADIUS OF A COXETER TRANSFORMATION

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Let Δ be a finite quiver (=oriented, connected graph) without oriented cycles. Let k be any field. The path algebra $k[\Delta]$ is a hereditary algebra, see [7]. The study of this kind of algebras had played a central role in the development of the Representation Theory of Algebras, see [6, 4, 13, 11].

For a representation X of $k[\Delta]$, we denote by $\underline{\dim}\, X=(\dim_k X(i))_{i\in\Delta_0}$ the dimension vector of X, where Δ_0 is the set of vertices of Δ. The *Coxeter matrix* ϕ_Δ satisfies

$$\underline{\dim}\,\tau X=(\underline{\dim}\, X)\phi_\Delta$$

where τX denotes the Auslander-Reiten translate of the non-projective indecomposable representation X. The *spectral radius* $\rho(\phi_\Delta)$ of the Coxeter matrix ϕ_Δ, contains relevant information about the behaviour of the translation τ, see [5, 11].

In this work, we consider some elementary relations between the spectral radii $\rho(\phi_{\bar\Delta})$ and $\rho(\phi_\Delta)$ for a *Galois covering* $\pi:\bar\Delta\to\Delta$. In particular, we show that for any covering $\pi:\bar\Delta\to\Delta$ defined by the action of a residually finite group and any finite subgraph F of $\bar\Delta$, we have $\rho(\phi_F)\leqq\rho(\phi_\Delta)$.

In [12], we have explored the relations between the spectral radii $r(\Delta)$ and $r(\bar\Delta)$ of the adjacency matrices $A_{\bar\Delta}$ and A_Δ, for a Galois covering $\pi:\bar\Delta\to\Delta$. In section 2, we show how to use these results to get some interesting bounds for $\rho(\phi_\Delta)$.

Finally, we get some applications. In relation with a problem posed by Kerner, we show that

$$\frac{g(\Delta)}{\rho(\phi_\Delta)}\leqq\frac{|\Delta_0|}{2},$$

where $g(\Delta)=|\Delta_1|-|\Delta_0|+1$ denotes the *genus* of the underlying graph of Δ.

1. Galois covering and Coxeter matrices.

1.1. Let n be the number of vertices of the quiver Δ.

Received October 14, 1991. Revised July 17, 1992.