TSUKUBA J. MATH.
Vol. 17 No. 1 (1993), 85–98

AN OPERATOR $L=aI-D_t^jD_x^{-j-\alpha}-D_t^{-j}D_x^{j+\alpha}$ AND ITS NATURE IN GEVREY FUNCTIONS

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1. Introduction.

In this paper, we shall study some spectral properties on the parameter $a\in\boldsymbol{C}$ of the following Goursat problem:

$$(1.1)\qquad \begin{aligned}&\{aD_t^lD_x^\beta-D_t^{l+j}D_x^{\beta-j-\alpha}-D_t^{l-j}D_x^{\beta+j+\alpha}\}u(t, x)=f(t, x),\\ &u(t, x)-w(t, x)=O(t^lx^\beta)\qquad(l\geqq 1,\ \beta\geqq 0),\end{aligned}$$

where $x, t\in\boldsymbol{C}$, $1\leqq j\leqq l$ and $-\beta-j\leqq\alpha\leqq\beta-j$.

In the case $\alpha=0$, the problem (1.1) was studied in an extremely precise form by Leary [1] and Yoshino [5, 6] in the space of holomorphic functions at the origin. Let $a=2\cos\pi\theta\in(-2, 2)$ $(0<\theta<1)$. Leray [1] introduced an auxiliary function $\rho(\theta)$ by

$$(1.2)\qquad \rho(\theta)=\liminf_{\boldsymbol{N}\ni h\to\infty}|\sin(h\pi\theta)|^{1/h}.$$

Here and in what follows, $\boldsymbol{N}$ and $\boldsymbol{Z}$ denotes the set of non negative integers and integers, respectively. They proved that the problem (1.1) is uniquely solvable in the space of holomorphic functions at the origin if and only if

$$(1.3)\qquad a\in\boldsymbol{C}\setminus(-2, 2)\quad\text{or}\quad a=\pm 2\quad\text{or}\quad \rho(\theta)>0.$$

Moreover, Leray-Pisot [2] proved that the set of zero points of $\rho(\theta)$ is uncountable with Lebesgue measure zero.

On the other hand, in the case $\alpha\neq 0$ the problem (1.1) is not solvable in the space of local holomorphic functions, and we have to study the problem in the space of formal or convergent power series with Gevrey estimate for the coefficients according as $\alpha>0$ or $\alpha<0$ (see Miyake-Hashimoto [4, Theorem B]). In this paper, we shall prove that the spectral properties on the parameter a distinguish the case $\alpha+j>0$ from the case $\alpha+j\leqq 0$. In the former case the meaning of the condition (1.3) will be understood clearly.

Received February 24, 1992, Revised May 28, 1992.