TSUKUBA J. MATH.
Vol. 17 No. 1 (1993), 77—84

RING EXTENSIONS AND ENDOMORPHISM RINGS OF A MODULE

By

Kazuhiko HIRATA and Yasukazu YAMASHIRO

In this paper we consider two conditions of a module related with a ring extension. The one is

$$(T)\quad M\otimes_S R \mathrel{<\!\!\!\oplus} M\oplus\cdots\oplus M$$

that is, $R\supset S$ is a ring extension and M a right R-module such that $M\otimes_S R$ is a direct summand of a finite direct sum of M as a right R-module. The second is

$$(H)\quad \mathrm{Hom}({}_QP,\ {}_QM) \mathrel{<\!\!\!\oplus} M\oplus\cdots\oplus M$$

that is, $P\supset Q$ is a ring extension and M a left P-module such that $\mathrm{Hom}({}_QP,\ {}_QM)$ is a direct summand of a finite direct sum of M as a left P-module. In § 1 we show that above two conditions are closely related with each other when $P=\mathrm{End}(M_S)$, $Q=\mathrm{End}(M_R)$ and when $R=\mathrm{End}({}_QM)$, $S=\mathrm{End}({}_PM)$, Propositions 1.1 and 1.2. In § 2 we apply the results in § 1 to H-separable extensions. We can give alternative proof of Sugano's theorem on H-separable extensions in [4]. It is easily seen that under the former condition (T) if M is a generator as an S-module then M is a generator as an R-module. Similarly we see that under the latter condition (H) if M is a Q-cogenerator then M is a P-cogenerator. But it seems too strong. In § 3 we treat about relative (co-)generators. Throughout this paper all rings have an identity, subrings contain this element, modules are unitary.

1. On conditions (T) and (H).

Let $R\supset S$ be a ring extension and M a right R-module. Let P and Q be the endomorphism rings of M as an S-module and as an R-module respectively, which operate on left side of M. Assume now the condition

$$(T)\quad M\otimes_S R \mathrel{<\!\!\!\oplus} M\oplus\cdots\oplus M.$$

Then there exist R-homomorphisms $f_i\colon M\otimes_S R\to M$ and $g_i\colon M\to M\otimes_S R$ such

Received February 19, 1992.