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IDEMPOTENT RINGS WHICH ARE EQUIVALENT TO RINGS WITH IDENTITY

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Let A be a ring such that $A=A^2$, but which does not necessarily have an identity element. In studying properties of the ring A through properties of its modules, it is pointless to consider the category A-MOD of all the left A-modules: for instance, every abelian group -with trivial multiplication- is in A-MOD. The natural choice for an interesting category of left A-modules seems to be the following: if a left A-module ${}_AM$ is *unital* when $AM=M$, and is *A-torsionfree* when the annihilator $t_M(A)$ is zero, then A-mod will be the full subcategory of A-MOD whose objects are the unital and A-torsionfree left A-modules. The category A-mod appears in a number of papers (for instance, [7-9]) and when A has local units [1, 2] or is a left s-unital ring [6, 12], then the objects of A-mod are the unital left A-modules. A-mod is a Grothendieck category and we study here the question of finding necessary and sufficient conditions on the ring A for A-mod to be equivalent to a category R-mod of modules over a ring with 1. This was already considered for rings with local units in [1], [2] or [3], and for left s-unital rings in [6]. Our situation is therefore more general.

In this paper, all rings will be associative rings, but we do not assume that they have an identity. A ring A has local units [2] when for every finite family $a_1, \cdots, a_n$ of elements of A there is an idempotent $e\in A$ such that $ea_j=a_j=a_je$ for all $j=1, \cdots, n$. A left A-module M is said to be unital if M has a spanning set (that is, if $AM=M$); and M has a finite spanning set when $M=\sum Ax_i$ for a finite family of elements $x_1, \cdots, x_n$ of M. The module ${}_AM$ will be called A-torsionfree when $t_M(A)=0$. A ring A is said to be left nondegenerate if the left module ${}_AA$ is A-torsionfree, and A is nondegenerate when it is both left and right nondegenerate (see [10, p. 88]). Clearly, a ring with local units is nondegenerate. The ring A will be called (left) s-unital [12] in case for each $a\in A$ (equivalently, for every finite family $a_1, \cdots, a_n$ of elements

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