

A REMARK ON FOLIATIONS ON A COMPLEX PROJECTIVE SPACE WITH COMPLEX LEAVES

Dedicated to Professor Hisao Nakagawa on his sixtieth birthday

By

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Introduction.

Let \mathcal{F} be a foliation on a Riemannian manifold M . The distribution on M which is defined to be orthogonal to \mathcal{F} is said to be normal to \mathcal{F} and denoted by \mathcal{F}^\perp .

Nakagawa and Takagi [8] showed that any harmonic foliation on a compact Riemannian manifold of non-negative constant sectional curvature is totally geodesic if the normal distribution is minimal. And successively the present author [2] proved a complex version of their result, that is, the above result holds also on a complex projective space with a Fubini-Study metric. However, recently, Li [4] pointed out a serious mistake in the proof of the result of Nakagawa and Takagi, and so of the author's. Therefore those results are now open yet.

On the other hand, Li [4] have studied a harmonic foliation on the sphere along the method of Nakagawa and Takagi, and obtained some interesting results.

The purpose of this paper is to give a complex analogue of the Li's results. Let $P_{n+p}(C)$ be the complex projective space with the Fubini-Study metric of constant holomorphic sectional curvature c . Let \mathcal{F} be a complex foliation on $P_{n+p}(C)$ with q -complex codimension and h the second fundamental tensor of \mathcal{F} . Then we shall prove the following;

THEOREM. *If the normal distribution \mathcal{F}^\perp is minimal, we have*

$$\int_{P_{n+p}(C)} S \left\{ \left(2 - \frac{1}{2p} \right) S - \frac{n+2}{2} c \right\} *1 \geq 0,$$

where S denotes the square of the length of h and $*1$ the volume element of $P_{n+p}(C)$.

COROLLARY. *Under the condition of the above theorem,*

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