

EXCEPTIONAL MINIMAL SURFACES WITH THE RICCI CONDITION

By

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0. Introduction.

Let $X^N(c)$ denote the N -dimensional simply connected space form of constant curvature c , and let M be a minimal surface in $X^N(c)$ with Gaussian curvature K ($\leq c$) with respect to the induced metric ds^2 . When $N=3$, M satisfies the Ricci condition with respect to c , that is, the metric $d\hat{s}^2 = \sqrt{c-K} ds^2$ is flat at points where $K < c$. Conversely, every 2-dimensional Riemannian manifold with Gaussian curvature less than c which satisfies the Ricci condition with respect to c , can be realized locally as a minimal surface in $X^3(c)$ (see [2]). Then it is an interesting problem to classify those minimal surfaces in $X^N(c)$ which satisfy the Ricci condition with respect to c , that is, to classify those minimal surfaces in $X^N(c)$ which are locally isometric to minimal surfaces in $X^3(c)$. In the case where $c=0$, Lawson [3] solved this problem completely. In [4] Naka (=Miyaoka) obtained some results in the case where $c>0$.

In [1] Johnson studied a class of minimal surfaces in $X^N(c)$, called exceptional minimal surfaces. In this paper, we discuss exceptional minimal surfaces in $X^N(c)$ which satisfy the Ricci condition with respect to c . Our results are as follows:

THEOREM 1. *Let M be an exceptional minimal surface lying fully in $X^N(c)$ where $c>0$. We denote by K the Gaussian curvature of M with respect to the induced metric ds^2 . Suppose that the metric $d\hat{s}^2 = \sqrt{c-K} ds^2$ is flat at points where $K < c$. Then either (i) $N=4m+1$ and M is flat, or (ii) $N=4m+3$.*

THEOREM 2. *Let M be an exceptional minimal surface lying fully in $X^N(c)$ where $c<0$. We denote by K the Gaussian curvature of M with respect to the induced metric ds^2 . Suppose that the metric $d\hat{s}^2 = \sqrt{c-K} ds^2$ is flat at points where $K < c$. Then $N=3$.*

REMARK. We note that every flat minimal surface in $X^N(c)$, where $c>0$,