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MINIMAL IMMERSION OF PSEUDO-RIEMANNIAN MANIFOLDS

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1. Preliminares.

Let E_q^n be the n-dimensional Pseudo-Euclidean space with metric tensor given by

$$g=-\sum_{i=1}^{q}(dx_i)^2+\sum_{j=q+1}^{n}(dx_j)^2$$

where $(x_1, x_2, \cdots, x_n)$ is a rectangular coordinate system of E_q^n. (E_q^n, g) is a flat Pseudo-Riemannian manifold of signature $(q, n-q)$.

Let c be a point in E_q^{n+1} (or E_{q+1}^{n+1}) and $r>0$. We put

$$S_q^n(c, r)=\{x\in E_q^{n+1} : g(x-c, x-c)=r^2\}$$

$$H_q^n(c, r)=\{x\in E_{q+1}^{n+1} : g(x-c, x-c)=-r^2\}.$$

It is known that $S_q^n(c, r)$ and $H_q^n(c, r)$ are complete Pseudo-Riemannian manifolds of signature $(q, n-q)$ and respective constant sectional curvatures r^{-2} and $-r^{-2}$. $S_q^n(c, r)$ and $H_q^n(c, r)$ are called the Pseudo-Riemannian sphere and the Pseudo-hyperbolic space, respectively. The point c is called the center of $S_q^n(c, r)$ and $H_q^n(c, r)$. In the following, $S_q^n(0, r)$ and $H_q^n(0, r)$ are simply denoted by $S_q^n(r)$ and $H_q^n(r)$, respectively. N_p^n denotes the Pseudo-Riemannian manifold with metric tensor of signature $(p, n-p)$. The Pseudo-Riemannian manifold, the Pseudo-Euclidean space, the Pseudo-Riemannian sphere and the Pseudo-hyperbolic space are simply denoted by the $P-R$ manifold, the $P-E$ space, the $P-R$ sphere and the $P-h$ space. The $P-R$ manifold N_1^n is called the Lorentz manifold and the $P-E$ space E_1^p is called the Minkowski space.

Let $f: M_p^m \to N_q^n$ be an isometric immersion of a $P-R$ manifold M_p^m in another $P-R$ manifold N_q^n. That is $f*\bar{g}=g$, where g and $\bar{g}$ are the indefinite metric tensors of M_p^m and N_q^n, respectively. $T(M_p^m)$ and $T^{\perp}(M_p^m)$ denote the tangent bundle and the normal bundle of M_p^m. ∇, $\bar{\nabla}$ and $\nabla^{\perp}$ denote the Riemannian connections and the normal connection on M_p^m, N_q^n and $T^{\perp}(M_p^m)$, respectively. Then for any vector fields $X, Y\in T(M_p^m)$, $v\in T^{\perp}(M_p^m)$, we have the Gauss formula

$$\bar{\nabla}_X Y=\nabla_X Y+B(X, Y),$$

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