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HARMONIC FOLIATIONS ON THE SPHERE

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Introduction.

Let M be a compact orientable manifold and let $\mathcal{F}$ be a harmonic foliation on M with respect to a bundle-like metric. Kamber and Tondeur [4] proved a fundamental formula for a special variation of $\mathcal{F}$, and making use of it they proved that the index of a harmonic foliation $\mathcal{F}$ on the sphere S^n $(n>2)$ for which the standard metric is bundle-like is not smaller than $q+1$, where q is the codimension of $\mathcal{F}$. On the other hand, Nakagawa and Takagi [6] proved that any harmonic foliation on a compact space form $M^n(c)$, $c\geqq 0$, for which the normal plane field is minimal is totally geodesic. Here a complete Riemannian manifold of constant curvature is called a *space form* and an n-dimensional space form of constant curvature c is denoted by $M^n(c)$. However a formula in [6] contains an error, and hence the above result is yet open.

The purpose of this paper is to study a harmonic foliation on the sphere. We use the method of Nakagawa and Takagi [6] to calculate the divergence of a vector field and obtain a formula of Simons' type. Then, after Chern, do Carmo and Kobayashi [2] it is proved that a harmonic foliation $\mathcal{F}$ of codimension q on an n-dimensional unit sphere satisfying $S\leqq(n-q)/(2-1/q)$ for which the normal plane field is minimal, is totally geodesic or $n=4$, $q=2$, where S denotes the square of the norm of the second fundamental form of each leaf. Moreover, was also prove that if $S\leqq(n-q)/(2-1/q)$ or $K\geqq(q-1)/(2q-1)$ for a harmonic foliation $\mathcal{F}$ of codimension q on the unit sphere with respect to a bundle-like metric, here K denotes the sectional curvature of leaves, then $\mathcal{F}$ is totally geodesic. Thus they have been completely classified by the theorem due to Escobales [3].

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