TSUKUBA J. MATH.
Vol. 15 No. 1 (1991), 219—226

COMPLETELY CELL SOLUBLE SPACES

(Dedicated to Professor Yukihiro Kodama on his 60th birthday)

By

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1. Introduction.

A topological space X is called homogeneous if for arbitrary points $x, y\in X$ there exists a homeomorphism f from X onto itself such that $f(x)=y$. Is every compact T_2-space the continuous image of a homogeneous compact T_2-space (Arhangel'skii [2])? Particularly, is a compact T_2-space nonhomogeneous if it can be mapped continuously onto βN (van Douwen [3])? These interesting problems remain unsolved. Related to these problems, Motorov showed that there exists a metrizable compact T_2-space which is not a retract of any homogeneous compact T_2-space. In the specific idea of Motorov, Arhangel'skii ([1], [2]) found an interesting topological property called cell solubility which every retract of an arbitrary homogeneous compact T_2-space posesses. He raised some problems related to this topological property. We solved already one of his problems [6]. In this paper we will answer to some other problems of Arhangel'skii.

2. Definitions.

The following definitions were introduced by Arhangel'skii [1], [2].

2.1. DEFINITION. Let X be a topological space. A map F of X into the set of all closed subsets of X is called a *cellularity* on X if the following conditions are satisfied:

1) $x\in F(x)$,

2) if $y\in F(x)$ then $F(y)\subset F(x)$,

3) if f is a homeomorphism from X onto itself such that $f(x)=y$ then $f(F(x))=F(y)$.

The sets $F(x)$ are called the *terms* of the cellularity F. A cellularity F on a space X is called *disjoint* if for any $x, y\in X$ either $F(x)=F(y)$ or $F(x)\cap F(y)$

Received December 14, 1989.