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ON TYPE NUMBER OF REAL HYPERSURFACES IN $P_n(C)$

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Introduction.

Let $P_n(\boldsymbol{C})$ denote an n-dimensional complex projective space with the Fubini-Study metric of constant holomorphic sectional curvature $4c$. Real hypersurfaces in $P_n(\boldsymbol{C})$ have been studied by many differential geometers (See [2], [3], [4], [5] and [6]).

In particular, as for a problem with respect to the typec number t, i.e., the rank of the second fundamental form of real hypersurfaces M in $P_n(\boldsymbol{C})$, R. Takagi showed in [6] that there is a point p on M such that $t(p)\geqq 2$, and M. Kimura and S. Maeda [4] gave an example of real hypersurfaces in $P_n(\boldsymbol{C})$ satisfying $t=2$, which is non-complete. In this paper we shall prove

THEOREM 1. *Let M be a complete real hypersurface in $P_n(\boldsymbol{C})(n\geqq 3)$. Then there exists a point p on M such that $t(p)\geqq 3$.*

REMAPK. It is known that a certain geodesic hypersphere in $P_2(\boldsymbol{C})$ has a property $t=2$ (cf. [2], [7]). Thus the assumption $n\geqq 3$ in Theorem 1 can not be removed.

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1. Preliminaries.

Let M be a real hypersurface in $P_n(\boldsymbol{C})(n\geqq 2)$. Let $\{e_1, \cdots, e_{2n}\}$ be a local field of orthonormal frame in $P_n(\boldsymbol{C})$ such that, restricted to M, $e_1, \cdots, e_{2n-1}$ are tangent to M. Denote its dual frame field by $\theta_1, \cdots, \theta_{2n}$. We use the following convention on the range of indices unless otherwise stated; $A, B, \cdots, =1, \cdots, 2n$ and $i, j, \cdots, =1, \cdots, 2n-1$.

The connection forms θ_{AB} are defined as the 1-forms satisfying

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