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THE DIRICHLET PROBLEM WITH L^p BOUNDARY DATA IN DOMAINS WITH DINI CONTINUOUS NORMAL

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In a series of articles [1-4, 6-8, 15, 16, 22-26], Mikhailov, Chabrowski, and others have used energy inequality methods to study the Dirichlet problem with L^p boundary values in C^2 domains. Here we show to adapt this method to $C^{1,\mathrm{Dini}}$ domains. Certain important elements of our analysis are present in work of Petrushko [26] for $C^{1,\alpha}$ domains, but our approach was motivated by different considerations. The key idea in the energy method (concerning boundary values) is that the traces of the solution of a suitable elliptic equation on "parallel" surfaces to the boundary should have a limit as these surfaces converge to the boundary. For C^2 domains this convergence is readily understood because there is a natural C^1 diffeomorphism between a level surface of the distance function (at least near the boundary) and the boundary itself. Petrushko provided special local diffeomorphisms between level surfaces of a regularized distance and the $(n-1)$ dimensional ball which fit together nicely. Here we use essentially any regularized distance and our conditions on the coefficients of the equation are weaker than Petrushko's.

A second approach to the Dirichlet problem with L^p boundary data is given by the methods of singular integrals. See [9-12, 17, 18]. This approach has the advantage of considering weaker regularity hypotheses on the leading coefficients of the elliptic operator and on the domain; however, none of the papers mentioned here considers lower order terms (in fact, several are concerned only with Laplace's equation) and some deep and subtle machinery is required. In addition a different definition of trace is used; the solution of the differential equation now approaches its boundary values nontangentially a.e. with respect to the elliptic measure induced by the differential equation. A key step, then, is the verification that this measure is absolutely continuous with respect to ordinary surface measure. Some connections between the two definitions of trace appear in [1].

An intermediate approach was recently proposed by Gushchin [14] to show

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