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# ON SOME PASTING CYLINDERS ONTO A MANIFOLD WITH NEGATIVE (RICCI, SCALAR) CURVATURE ALONG COMPACT BOUNDARIES

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## 1. Introduction.

In Topology two manifolds are often pasted together. If both of these manifolds are riemannian, it is natural to consider the relation between geometry of the obtained manifold and geometry of original two manifolds. For this problem L. Z. Gao showed the following result [5]. If $M$ and $N$ are two complete riemannian manifolds with negative Ricci curvature of same dimension, then there is a complete riemannian metric on the connected sum $M\#N$ with negative Ricci curvature. Of course when he constructs this metric on $M\#N$, he changes both the metrics on $M$ and on $N$. Changing a point of view, we would like to take $M$ with boundary $\partial M$ and paste $M$ to another manifold with boundary along $\partial M$ *not* changing the metric of $M$. In this paper we show the following. Let $M$ be a negatively Ricci (resp. scalar) curved complete manifold with compact boundary and $\check{M}$ be a manifold without boundary obtained by pasting cylinders onto $M$. Then there is a complete metric on $\check{M}$ with negative Ricci (resp. scalar) curvature such that the inclusion $M\subset\check{M}$ is an isometric embedding and this metric is a warped product metric (cf. [2]) on $\check{M}\setminus S$ (where $S$ is an open neighborhood of $M$) whose vertical fibers are homothetic to $\partial M$. Moreover if we assume that $\partial M$ is totally geodesic in $M$, we have the same assertion as above for sectional curvature. Manifolds with metrics of this type are studied in some papers (e. g. [1], [4]).

We use the following notation for a $C^\infty$ manifold $X$ with or without boundary,

$\mathrm{Sec}_-(X):=\{$all riemannian metrics on $X$ with negative sectional curvature$\}$,

$\mathrm{Ric}_-(X):=\{$all riemannian metrics on $X$ with negative Ricci curvature$\}$,

$\mathrm{Sca}_-(X):=\{$all riemannian metrics on $X$ with negative scalar curvature$\}$.

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